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Nonlinear taxation of income and education in the presence of income-misreporting

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Abstract

We study the joint design of nonlinear income and education taxes when the government pursues redistributive objectives. A key feature of our setup is that the ability type of an agent can affect both the costs and benefits of acquiring education. Market remuneration of agents depends on both their innate ability type and their educational choices. Our focus is on the properties of constrained efficient allocations when educational choices are publicly observable at the individual level, but earned income is subject to misreporting. We find that income-misreporting (IM) affects the optimal distortions on income and education and shed light on the reasons for it and mechanisms through which it is done. We show how and why IM strengthens the case for downward distorting the educational choices of low-ability agents. Finally, we find that IM provides another mechanism that makes commodity taxation useful.

JEL classification: H21, H26, J31.

Keywords: optimal taxation; education; human capital; income-misreporting; redistribution.

1 Introduction

The contributions growing out of Mirrlees' (1971) seminal paper on optimal income taxation have mostly assumed that an individual's productivity or wage rate is exogenously given. More recently, however, a comparatively small literature has analyzed optimal redistributive taxation in settings with endogenous wages. These contributions may be divided roughly into three strands. One strand maintains the assumption of perfect competition in the labor market and generates wage endogeneity by treating workers of different skill type as separate inputs that are imperfectly substitutable in production function.[1] A second strand generates wage endogeneity by introducing frictions in the labor market that may either be due to imperfect competition or to problems of asymmetric information between workers and employers.[2] The third strand endogenizes wages by allowing for the possibility to invest in productivity-enhancing education.[3]

Within this last strand, a number of contributions assume that educational attainment is publicly observable at the individual level and thus can be taxed nonlinearly. These studies have greatly enhanced our understanding of what determines the direction of the optimal distortion on educational choices of agents. Yet, while they differ in many aspects, they all maintain the Mirrleesian assumption of public observability of earnings. This makes nonlinear taxation of both incomes and educational expenditures possible. However, in reality, income-misreporting is often a relevant phenomenon—a fact that might very well undermine the efficacy of the income tax in achieving redistribution.

The distinctive feature of our contribution lies in the recognition that agents can conceal part of their earned income for tax purposes. Our main goal is to investigate if, and how, the optimal distortion on agents' educational choices varies depending on whether or not earned income is perfectly observable by the government at the individual level. And, to simplify our analysis, we model income-misreporting (hereafter, IM) following the so called riskless approach pioneered by Usher (1986).

As a vehicle for our study, we set up a two-type optimal income tax model (à la Stern (1982) and Stiglitz (1982)) where an agent's productivity depends on his type (innate ability), and on the amount of education he acquires. To attain a given amount of education, agents incur an effort cost which is type-dependent (in addition to monetary cost of education).[4] We

[1] See, e.g., Stiglitz (1982, 1987), Allen (1987), Naito (1999), Pirttilä and Tuomala (2002), Blackorby and Brett (2004), Gaube (2005), Gahvari (2014).

[2] See, e.g., Aronsson and Sjögren (2003, 2004), Hungerbühler *et al.* (2006), Kessing and Konrad (2006), Bastani, Blumkin and Micheletto (2015), Aronsson and Micheletto (2021).

[3] See, e.g., Ulph (1977), Tuomala (1986), Brett and Weymark (2003), Bovenberg and Jacobs (2005), Blumkin and Sadka (2008), da Costa and Severo (2008), Maldonado (2008), Jacobs and Bovenberg (2011), Guo and Krause (2013), Findeisen and Sachs (2016, 2018), Stantcheva (2017).

[4] The idea of an agent's "type" as a multi-dimensional characteristic which affects both a person's productivity and a person's cost of attaining a given education level resonates with the concluding remarks in Hellwig (2008, p. 8): "... it makes sense to think of both the productivity of an agent with education level e and the cost of achieving this education level as being unobservable, determined by one or several hidden characteristics. The

characterize the properties of an informationally constrained Pareto-efficient tax policy, focusing on the so-called “normal” case where the direction of redistribution goes from the high- to the low-ability type.

Our main results can be summarized as follows. First, when an agent’s productivity or wage depends only on his education level and not *directly* on his type (i.e., the effect of one’s type is channeled only indirectly through the education level he chooses), IM does not affect the qualitative properties of an optimal tax policy. Whether earned income is perfectly observable at the individual level or not, all agents—high- and low-ability alike—face a zero marginal income tax rate. Tax treatment of educational attainment, on the other hand, is not the same. Its choice is left undistorted for high-ability agents but downward distorted for low-ability agents.

Second, in the more general case wherein an agent’s productivity or wage depends *directly* on both his type and educational attainment, IM does not change the characterization and the sign of the marginal income tax rate faced by low-ability agents (which should be positive), and high-ability agents (which should be zero). It also leaves unscathed the result that education should be undistorted for high-ability agents. However, compared with a setting where earned income is perfectly observed by the government at the individual level, it strengthens the case for a downward distortion on the educational choice of low-ability agents. We will explore the various factors that are behind this later in the paper. However, it is worth pointing out here that, with or without IM, education provides another instrument for making “mimicking” less attractive (making the preferred choice of the low-ability agents less desirable to high-ability agents), thus allowing more redistribution to low-ability agents.

Third, in the general case wherein an agent’s productivity depends directly on his type, IM implies that consumption taxation is no longer a redundant policy instrument. We thus reconsider our results by assuming that a linear consumption tax is used alongside a joint nonlinear tax on education and reported income. Under this scenario, it becomes desirable to let high-ability agents face a negative marginal tax on reported income and also to distort upwards their educational choice. For low-ability agents, consumption taxation exerts a moderating effect on the optimal marginal tax on reported income. It also generates a mitigating effect on the tendency, attributable to the possibility of IM, to warrant a downward distortion on the educational choice of low-ability individuals.

The paper is related to a diverse body of literature in public economics.

(i): Tax evasion. Most of the early evasion literature, following the seminal contribution by Allingham and Sandmo (1972), assumes that decisions about income-misreporting involve risk. Evasion may be detected with some probability, for instance due to random audits by the tax

question then should be how these two information problems interact and how this interaction affects optimal utilitarian taxation. Recognizing this as an issue provides a good basis for further research on the respective roles of heterogeneities in productivities and in education costs".

authorities, in which case a sanction applies (see, e.g., Cremer, Marchand and Pestieau, 1990, Cremer and Gahvari, 1994, 1996, Schroyen, 1997). The riskless approach to evasion, introduced in the literature by Usher (1986), assumes that taxpayers are able to fully avoid detection by incurring a cost that depends on the amount they misreport. The cost function may be implicitly assumed to capture some of the elements from the uncertainty model, for instance to make the cost higher the more extensive is the auditing activity of the tax collector. As in the uncertainty case, there is a trade-off between the gain from lowering the tax by IM and the cost incurred, which is modeled as a pure concealment cost. Following the contribution by Usher (1986), the riskless approach has been used in a number of subsequent contributions (e.g., Mayshar, 1991, Boadway, Marchand and Pestieau, 1994, Kopczuk, 2001, Slemrod, 2001, Christiansen and Tuomala, 2008, Chetty, 2009, Gahvari and Micheletto, 2014, Gerritsen, 2021).

(ii) Redistributive role of education policy. Beginning with Arrow (1971), a large body of public economics literature has investigated the redistributive role of education policy. Earlier contributions, including Arrow (1971), Green and Sheshinski (1975) and Bruno (1976), left aside asymmetric information problems and assumed that the government could observe an individual's type. The main goal of these contributions was to characterize the optimal allocation of a given amount of educational expenditure amongst a population of individuals of different ability. Considering the educational policy in isolation, or assuming an exogenously given income tax schedule, these papers could not shed light on the relative merits of tax- and expenditure policy for redistributive purposes.

(iii) Interaction of income redistribution and educational policy. Ulph (1977) and Hare and Ulph (1979) developed Bruno's work on the interaction of income redistribution and educational policy by allowing both types of policies to be simultaneously optimized. However, they retained the assumption that the ability to benefit from education is observed by the education authorities. Relaxing this assumption, and assuming that a nonlinear income tax is the only government's policy instrument, Tuomala (1986) analyzed how individuals' educational choices affect the progressivity of the optimal nonlinear income tax. He did so in the context of a timeless model where private agents only differ in their ability to transform education into labor productivity. Subsequently, Brett and Weymark (2003) generalized Tuomala's model to a setting where individuals differ both in terms of their ability to transform education into labor productivity, and in terms of the time needed to acquire a given amount of education. However, as in Tuomala's case, they assumed that taxes could only be set as a function of earned income.

Bovenberg and Jacobs (2005) was the first paper which jointly optimized, within a Mirrleesian static framework, a nonlinear tax on income and on educational expenditure; they also considered the case where education entails for agents both a resource- and an effort-cost, but assumed away the possibility that agents differ in the effort cost of acquiring education. A famous result that they obtained was that education subsidies and income taxes are "Siamese twins", in that education subsidies should be used for the sole purpose of offsetting the distortions created by redistributive income taxation. This result was later challenged by Maldonado (2008) who showed that distorting the educational choices of agents is desirable when the education elasticity of wage varies with ability.[5]

(iv) Dynamic Mirrleesian settings. From a normative standpoint, the interaction between the taxation of income and education has also been investigated in dynamic Mirrleesian settings where commitment issues or risky properties of human capital investment have been important elements of the analysis. The differences between an optimal policy under full or limited commitment have been studied by Guo and Krause (2013) and Findeisen and Sachs (2018). Uncertainty about the return to human capital investment implies that the tax policy serves a dual purpose, achieving redistribution and providing insurance. The role of uncertainty has been analyzed in various contributions such as da Costa and Maestri (2007), da Costa and Severo (2008), Findeisen and Sachs (2016), Stantcheva (2017) and Kapicka and Neira (2019). The first three papers consider two-period models where agents make a one-shot education decision, with a one-time realization of uncertainty. Stantcheva (2017) considers an n-period model where investment in education occurs during the entire life-cycle of an agent, with progressive realization of uncertainty throughout life. Kapicka and Neira (2019) consider a two-period model with one-time realization of uncertainty but assume that human capital investments are only partially observable by the planner.

Finally, in finding another avenue for making consumption taxation a useful instrument for optimal tax policy when there is IM, the paper contributes to the large literature that the celebrated Atkinson-Stiglitz (1976) paper on the redundancy of commodity taxation has spawned. Many authors have found different mechanisms to justify commodity taxation while retaining the Atkinson-Stiglitz separability assumption. The following is a partial list of factors considered. Differential public observability: Boadway, Marchand, and Pestiau (1994), Cremer, Pestiau, and Rochet (2002); Uncertainty and commitment: Cremer and Gahvari (1995a, 1995b, 1999); Heterogeneity in tastes: Cremer and Gahvari (1998, 2002), Saez (2002), Boadway et al. (2002); Heterogeneity in endowment: Boadway, Marchand and Pestieau (2000), Cremer, Pestieau, and Rochet (2001); Differential consumption time: Boadway and Gahvari (2006), Gahvari (2007); Household production: Cremer and Gahvari (2015); Market sophistication: Gahvari and Micheletto (2016).

The rest of the paper is organized as follows. Section 2 describes the behavior of the agents under laissez-faire. Section 3 presents the income-misreporting technology that is available to

[5] Later on, Jacobs and Bovenberg (2011) extended Maldonado's model by considering a more general earnings function which allowed for the possibility that the education elasticity of earnings depended both on innate ability and labor supply.

agents, sets up the government's problem, and characterizes the properties of an optimal tax policy that jointly depends on the amount of education acquired by an agent and his/her reported income. Section 4 adds a linear consumption tax to the armoury of instruments available to the government. Section 5 discusses the robustness of our results to alternative assumptions regarding the wage function. Section 6 offers concluding remarks.

2 The setting

Agents differ in ability type, which is described by the parameter θ. A higher-ability corresponds to a higher value of θ. The labor productivity of a given agent depends on his type θ and education level e. Attaining a given level of education entails both a resource- and an effort-cost. The resource cost is given by p (per unit of education e) and the effort cost is captured by a function $\varphi(\theta, e)$ which in general depends on the individual's ability type.

An agent of ability θ who acquires education in the amount e has labor productivity $w(\theta, e)$, which means that he supplies $w(\theta, e)$ units of labor in efficiency units per unit of time. We assume that $\partial w(\theta, e)/\partial\theta \geq 0$ and $\partial w(\theta, e)/\partial e > 0$. There is a single consumption good, denoted by c, and produced using labor in efficiency units as the sole input. The technology exhibits constant returns to scale. The consumption good is treated as the numéraire and we choose the units of measurement so that one unit of labor in efficiency units produces one unit of output. The labor market is perfectly competitive, so that an individual's wage is equal to the marginal (and average) product of his labor $w(\theta, e)$. Earned income, denoted by I, is thus given by $I \equiv w(\theta, e) L$, where L is the amount of labor supplied to the market.

Agents' preferences are described by the function

$$U = u(c) - v(L) - \varphi(\theta, e), \tag{1}$$

where $u'(c) > 0$, $u''(c) \leq 0$, $v'(L) > 0$, $v''(L) > 0$, $\partial\varphi(\theta, e)/\partial\theta \leq 0$, $\partial\varphi(\theta, e)/\partial e > 0$, $\partial^2\varphi(\theta, e)/\partial e\partial e > 0$ and $\partial^2\varphi(\theta, e)/\partial e\partial\theta \leq 0$. Under laissez-faire an individual solves the following maximization problem:

$$\max_{I,e} \quad u(I - pe) - v\left(\frac{I}{w(\theta, e)}\right) - \varphi(\theta, e).$$

The associated first-order conditions with respect to I and e are given by:

$$\frac{v'}{u'} = w, \tag{2}$$

$$\frac{L}{w}\frac{\partial w}{\partial e}\frac{v'}{u'} - \frac{\partial\varphi/\partial e}{u'} = p, \tag{3}$$

where the LHS of (2) represents the marginal rate of substitution between labor and consumption (MRS_{Lc}), and the LHS of (3) represents the marginal rate of substitution between education and consumption, or as it is called below, the marginal willingness to pay for education (MWP_{ec}).

3 Fully nonlinear taxation with income-misreporting

3.1 Design

Consider a discrete setting with two types of agents, those with $\theta = \theta^\ell$ (low-ability) and those with $\theta = \theta^h$ (high-ability), with $\theta^h > \theta^\ell$. The government intends to design a Pareto-efficient tax policy that allows achieving some given revenue-raising- and redistributive goals. The informational structure of the problem includes the standard Mirrleesian assumption that the government knows the distribution of types in the population but does not know "who is who". This rules out the possibility of using first-best, type-specific, lump-sum taxes/subsidies. However, in contrast to what is commonly assumed in optimal taxation models, we shall assume that earned incomes are not publicly observable either. This opens up the possibility of tax evasion through IM. The educational level achieved by a given individual, on the other hand, is assumed publicly observable and thus taxable.

To model IM, we follow the riskless approach introduced by Usher (1986); specifically, once agents have incurred a cost, they face no risk of detection. This simple structure allows the government to achieve its objectives through a general (nonlinear) tax function $T(\cdot, \cdot)$ which depends on *reported income*, M, and education level e. To see this, recall that in a two-group model without IM, one needs only determine the two groups' allocations. This can be done by a direct mechanism consisting of two bundles, each specifying a particular amount of income (earned equal to reported), education, and consumption. The same procedure works in our model if the two bundles, the one intended for the low-skilled agents and the other for the high-skilled ones, are specified in terms of reported income M, education e, and tax payment T (equivalently, net-of-tax reported income: $B \equiv M - T$). The reason is that, as we show below, an (e, M, B)-bundle corresponds to an (e, L, c)-bundle; notwithstanding the fact that reported incomes will likely differ from actual earned incomes.

Consider the optimization problem of an agent who is to choose between bundles (e^ℓ, M^ℓ, B^ℓ) and (e^h, M^h, B^h). He will choose the bundle that maximizes his utility (1). Denote by a the difference between earned- and reported-income. IM is costly: a taxpayer who evades a incurs a (pecuniary) cost $\sigma(a)$ where $\sigma(\cdot)$ is assumed to be non-negative, increasing in (the absolute value of) a, strictly convex. We also assume $\sigma(0) = \sigma'(0) = 0$. The consumption level of a taxpayer who selects (e, M, B), and subsequently misreports a, is equal to

$$c = B + a - \sigma(a), \tag{4}$$

where we have dropped the superscripts for ease in notation. Observe that in designing this mechanism, we have implicitly assumed that the pecuniary cost of education is paid by the government. Alternatively, one may assume the cost is incurred by the individual paying p per unit of e. The two formulations provide identical results.[6]

With the taxpayer's true earnings being equal to $w(\theta, e) L$, we have

$$L = \frac{M + a}{w(\theta, e)}. \tag{5}$$

Substituting (4) and (5) in (1), yields

$$U = u(B + a - \sigma(a)) - v\left(\frac{M + a}{w(\theta, e)}\right) - \varphi(\theta, e). \tag{6}$$

The taxpayer's optimization problem consists of choosing a to maximize (6). This results in the first-order condition

$$v'\left(\frac{M + a}{w(\theta, e)}\right) = w(\theta, e)\left[1 - \sigma'(a)\right] u'(B + a - \sigma(a)), \tag{7}$$

which determines a and subsequently c. Obviously, given (5), choosing a is tantamount to choosing L.

Having shown that a taxpayer's choice of (e, M, B) determines his consumption bundle (e, L, c), we now discuss how the mechanism designer determines (e^ℓ, M^ℓ, B^ℓ) and (e^h, M^h, B^h). There are two types of constraints that must be considered in this problem. One is that the bundles must satisfy the economy's resource constraint. The other, with the taxpayers being ultimately free to determine their allocations when given a tax function, is that the bundles must be incentive-compatible. This requires that agents do not behave as "mimickers", misrepresenting their ability type: each agent must weakly prefer the (e, M, B)-bundle intended for his ability type to that intended for the other type.

For a given (e, M, B)-bundle, the (conditional) indirect utility of an agent of type θ is given by

$$V(e, M, B; \theta) \equiv u(B + a^* - \sigma(a^*)) - v\left(\frac{M + a^*}{w(\theta, e)}\right) - \varphi(\theta, e), \tag{8}$$

where a^* represents the value of a that solves the first-order condition (7). Normalizing to one the size of the total population and denoting by π the proportion of agents of type ℓ (low-ability), the government's problem (hereafter, problem $\mathcal{P}1$) can be formalized as

$$\max_{e^\ell, M^\ell, B^\ell, e^h, M^h, B^h} \quad V\left(e^\ell, M^\ell, B^\ell; \theta^\ell\right)$$

subject to:

$$\begin{aligned}
&V\left(e^h, M^h, B^h; \theta^h\right) \geq \overline{V}, \\
&V\left(e^h, M^h, B^h; \theta^h\right) \geq V\left(e^\ell, M^\ell, B^\ell; \theta^h\right), \\
&\pi\left(M^\ell - B^\ell - pe^\ell\right) + (1 - \pi)\left(M^h - B^h - pe^h\right) \geq \overline{R},
\end{aligned}$$

[6] Notice that any given (e, M, B)-bundle where the resource cost of education is paid for by the government is equivalent, from an agent's standpoint, to a $(e, M, \widetilde{B})$-bundle where $\widetilde{B} \equiv B + pe$ and the resource cost of education is paid for by the agent. The two bundles, under the respective assumptions about who pays for the resource cost of education, are also equivalent in terms of net revenue collected by the government.

where $\overline{V}$ and $\overline{R}$ represent, respectively, an exogenous pre-specified utility level for agents of type h (high-ability) and a government's exogenous revenue requirement.[7] Notice that, according to the last constraint, the resource cost of education is covered by the government. As pointed out in footnote 6, this kind of approach is without loss of generality.

Denoting the Lagrange multipliers associated with the first, second, and third constraint by δ, λ and μ, the optimal values of $e^h, M^h, B^h, e^\ell, M^\ell, B^\ell$ are determined by the first-order conditions to this problem presented in the Appendix A [equations (A1)-(A6)].

3.2 Properties

We can now specify the properties of the tax schedule $T(M,e)$ that implements (e^j, M^j, B^j), and thus (e^j, L^j, c^j), for $j = \ell, h$. Faced with the schedule $T(M,e)$, or alternatively $B(M,e) \equiv M - T(M,e)$, an agent solves the following maximization problem

$$\max_{M,e,a} \quad u\left(M + a - T(M,e) - \sigma(a)\right) - v\left(\frac{M+a}{w(\theta,e)}\right) - \varphi(\theta,e).$$

From the first-order conditions of the above problem, we can derive the following implicit characterizations for the marginal tax rates $T_M \equiv \partial T(M,e)/\partial M$ and $T_e \equiv \partial T(M,e)/\partial e$):

$$T_M \equiv \frac{\partial T(M,e)}{\partial M} = 1 - \frac{v'}{wu'}, \tag{9}$$

$$T_e \equiv \frac{\partial T(M,e)}{\partial e} = \frac{(M+a)\,\partial w/\partial e}{w^2}\frac{v'}{u'} - \frac{\partial\varphi/\partial e}{u'} = \frac{L}{w}\frac{\partial w}{\partial e}\frac{v'}{u'} - \frac{\partial\varphi/\partial e}{u'}. \tag{10}$$

Notice also that, combining (7) and (9), one gets that

$$T_M = \sigma'. \tag{11}$$

For later purposes, it is useful to define the marginal rate of substitution between M and B, denoted by MRS_{MB}, for an agent choosing a given (e, M, B)-bundle. From (8), by invoking the envelope theorem, we have:

$$\partial V(e,M,B;\theta)/M = -v'\left(\frac{M+a^*}{w(\theta,e)}\right)/w(\theta,e);$$

$$\partial V(e,M,B;\theta)/\partial B = u'\left(B + a^* - \sigma(a^*)\right).$$

Thus, we can define MRS_{MB} as

$$MRS_{MB} \equiv -\frac{\partial V(e,M,B;\theta)}{\partial M}\Big/\frac{\partial V(e,M,B;\theta)}{\partial B} = \frac{v'\left(\frac{M+a^*}{w(\theta,e)}\right)}{w(\theta,e)\,u'\left(B + a^* - \sigma(a^*)\right)}. \tag{12}$$

[7] In setting up the government's problem we have neglected to take into account the self-selection constraint requiring low-ability agents not to mimic high-ability agents. This approach can be justified by assuming that we focus on the so called "normal" case when the intended direction of redistribution is from the high- to the low-ability agents. Put differently, we are implicitly assuming that the value for $\overline{V}$ appearing on the RHS of the first constraint is lower than the utility enjoyed by high-ability agents under laissez-faire.

Based on (12), we introduce the following notations for the marginal rates of substitution between M and B for a j-type (with $j = h, \ell$) and for an h-type mimicking an ℓ-type,

$$MRS_{MB}^{j} = \frac{v'\left(\frac{M^j + a^j}{w(\theta^j, e^j)}\right)}{w\left(\theta^j, e^j\right) u'\left(B^j + a^j - \sigma\left(a^j\right)\right)} = \frac{v'\left(\frac{M^j + a^j}{w(\theta^j, e^j)}\right)}{w\left(\theta^j, e^j\right) u'\left(c^j\right)}, \tag{13}$$

$$MRS_{MB}^{h\ell} = \frac{v'\left(\frac{M^\ell + a^{h\ell}}{w(\theta^h, e^\ell)}\right)}{w\left(\theta^h, e^\ell\right) u'\left(B^\ell + a^{h\ell} - \sigma\left(a^{h\ell}\right)\right)} = \frac{v'\left(\frac{M^\ell + a^{h\ell}}{w(\theta^h, e^\ell)}\right)}{w\left(\theta^h, e^\ell\right) u'\left(c^{h\ell}\right)}, \tag{14}$$

where $a^j \equiv a^*\left(e^j, M^j, B^j; \theta^j\right)$, for $j = h, \ell$, and $a^{h\ell} \equiv a^*\left(e^\ell, M^\ell, B^\ell; \theta^h\right)$.

We are now in a position to give a characterization for the implicit marginal tax rates faced by high- and low-ability agents at an optimum. This is done in Proposition 1 where we denote the earnings of a low-ability agent by $I^\ell \equiv M^\ell + a^\ell$ and the earnings of a high-ability agent behaving as a mimicker by $I^{h\ell} \equiv M^\ell + a^{h\ell}$.

Proposition 1 *Consider a two-group optimal income tax model with IM, wherein a worker's productivity depends directly on his ability type and educational attainment. The optimal implicit marginal tax rates faced by the high- and low-ability agents are given by:*

$$T_M\left(M^h, e^h\right) = 0, \quad T_e\left(M^h, e^h\right) = p, \tag{15}$$

and

$$T_M\left(M^\ell, e^\ell\right) = \frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left(MRS_{MB}^{\ell} - MRS_{MB}^{h\ell}\right), \tag{16}$$

$$\begin{aligned} T_e\left(M^\ell, e^\ell\right) &= p + \frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left(I^{h\ell}\epsilon_{w,e}^{h\ell} MRS_{MB}^{h\ell} - I^\ell \epsilon_{w,e}^{\ell} MRS_{MB}^{\ell}\right) \\ &\quad + \frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\frac{\partial\varphi\left(\theta^\ell, e^\ell\right)/\partial e^\ell}{u'\left(c^\ell\right)} - \frac{\partial\varphi\left(\theta^h, e^\ell\right)/\partial e^\ell}{u'\left(c^{h\ell}\right)}\right], \end{aligned} \tag{17}$$

where $\epsilon_{w,e}^{h\ell}$ and $\epsilon_{w,e}^{\ell}$ denote the semi-elasticity of the wage rate with respect to education for, respectively, a high-ability mimicker and a low-ability type, i.e.

$$\epsilon_{w,e}^{h\ell} \equiv \frac{\partial w\left(\theta^h, e^\ell\right)}{\partial e^\ell} / w\left(\theta^h, e^\ell\right) \ \text{ and } \ \epsilon_{w,e}^{\ell} \equiv \frac{\partial w\left(\theta^\ell, e^\ell\right)}{\partial e^\ell} / w\left(\theta^\ell, e^\ell\right). \tag{18}$$

Proof. See Appendix A. ■

The fact that $T_M\left(M^h, e^h\right) = 0$ implies that the labor supply of high-ability agents is undistorted (which also implies that $a^h = 0$) and the fact that $T_e\left(M^h, e^h\right) = p$ tells us that the optimal marginal tax on education is given by a purely non-distortionary term that is meant to let agents internalize the marginal resource cost of e.[8] These particular results are not surprising

[8] Recall that agents do not *directly* pay for the resource costs of education; they pay for these costs *indirectly* through taxation. Rather than entering the private budget constraint (4), the resource costs of education appear in the government's revenue constraint (the last constraint of problem $\mathcal{P}1$). See Blomquist, Christiansen and Micheletto (2010) for more details on the distinction between distortionary and non-distortionary marginal tax rates.

as they are yet another reflection of the well-known "no distortion at the top" result, and an artifact of the assumption that redistribution is from high- to low-ability types. What is more interesting is the nature of the marginal tax rates faced by the low-ability types to which we now turn.

To proceed on this front, we find it useful to distinguish between two cases. Case (a): One's type does not affect his productivity *directly*; educational attainment serves as the only vehicle for any effect that one's type may have on his productivity (through the type-dependent effort-cost of achieving a given education level, namely, the function $\varphi(\theta, e)$) . Algebraically, this is depicted by $\partial w(\theta, e)/\partial\theta = 0$. Case (b): One's type has a positive direct effect on productivity separate from the effect channeled through education so that $\partial w(\theta, e)/\partial\theta > 0$.

3.3 Case (a): $\partial w(\theta, e)/\partial\theta = 0$

We start by presenting a lemma which will help in studying this case.

Lemma 1 *Assume that $\partial w(\theta, e)/\partial\theta = 0$. Then:*

(i) A high-ability worker, when behaving as a mimicker, will misreport the same amount as a low-ability worker. That is, $a^{h\ell} = a^{\ell}$.

(ii) For any given (e, M, B)-bundle we have that $MRS^{h\ell}_{MB} = MRS^{\ell}_{MB}$.

(iii) $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e}$.

Proof. See Appendix A. ■

The result in (i) is due to the fact that our riskless modeling of evasion, with a type-independent IM-cost function $\sigma(\cdot)$, ensures that an individual's choice of a is independent of his type for any given (e, M, B)-bundle; see the first-order condition (7). The result in (ii) follows because $a^{h\ell} \neq a^{\ell}$ serves as the only potential source for $MRS^{h\ell}_{MB}$ to differ from MRS^{ℓ}_{MB}. The result in (iii) comes from the definitions in (18) when w depends only on e.

Using the results of Lemma 1 in equations (16)–(17) simplifies them into

$$T_M\left(M^{\ell}, e^{\ell}\right) = 0, \tag{19}$$

$$T_e\left(M^{\ell}, e^{\ell}\right) = p + \frac{\lambda}{\mu\pi}\left[\frac{\partial\varphi(\theta^{\ell}, e^{\ell})}{\partial e^{\ell}} - \frac{\partial\varphi(\theta^{h}, e^{\ell})}{\partial e^{\ell}}\right], \tag{20}$$

which implies that $T_e\left(M^{\ell}, e^{\ell}\right) > p$ under the assumption that $\partial^2\varphi(\theta, e)/\partial e\partial\theta < 0$. The key to understand these results is to notice that, when taxes can be conditioned on education, a high-ability mimicker is denied any information rent associated with differences in productivities. This is due to the fact that, conditional on education, productivity is not type-dependent when $\partial w(\theta, e)/\partial\theta = 0$. Absent a difference in market productivities, there is no mimicking-deterring benefit from distorting the labor supply of low-ability agents; it would serve no screening purpose given that $MRS^{\ell}_{MB} = MRS^{h\ell}_{MB}$. This observation accounts for the result provided by eq. (19). Moreover, given that from Proposition 1 we also have that $T_M\left(M^h, e^h\right) = 0$, it also follows that, once taxes can be conditioned on education, it is useless to condition the tax liability also on reported income; a nonlinear tax on education is all that the government needs. High-ability mimickers only enjoy an information rent that is associated with differences in the (effort) cost of acquiring education, and this information rent is reflected in the term within square brackets in eq. (20). This term calls for setting the marginal tax on education at a level higher than p, entailing a downward distortion on e^{ℓ} which is justified by mimicking-deterring considerations. The intuition comes from the observation that, since the marginal (effort) cost of acquiring education is lower for a mimicker than for a low-ability agent, the marginal willingness to pay for education is higher for the former than for the latter.[9] Hence, introducing a small distortion on e^{ℓ} imposes a first-order utility loss on the mimicker, thereby relaxing the binding self-selection constraint, while at the same time exerting only a second-order effect on low-ability agents.

The above results, and in particular the redundancy of conditioning the tax liability also on reported income, warrant an important remark. This result hinges on the fact that for any given (e, M, B)-bundle, $a^{h\ell} = a^{\ell}$ when $\partial w(\theta, e)/\partial\theta = 0$, an outcome that descends from the assumption that there is no heterogeneity in the misreporting technology available to taxpayers. With heterogeneous IM-technologies, parts (i) and (ii) of Lemma 1 would cease to be valid.[10,11] As a consequence, if type-ℓ agents had access to a cheaper IM-technology, the government would benefit from conditioning the tax liability also on reported income, and letting $T_M\left(M^{\ell}, e^{\ell}\right)$ deviate from zero.[12] Intuitively, if IM is easier for those agents who are regarded as "more deserving" from a social point of view, there are redistributive gains from designing the tax schedule in such a way that transfer-recipients are induced to misreport their income. These

[9] As we pointed out at the end of Section 2, the expression on the LHS of (3) represents the marginal willingness to pay for education. Therefore, at a given (e, M, B)-bundle, MWP_{ec} can be equivalently re-expressed as $(M + a)\,\epsilon_{w,e} MRS_{MB} - (\partial\varphi/\partial e)/u'$. Applying the results provided by Lemma 1, we have that $\left(M + a^{h\ell}\right)\epsilon^{h\ell}_{w,e} MRS^{h\ell}_{MB} = \left(M + a^{\ell}\right)\epsilon^{\ell}_{w,e} MRS^{\ell}_{MB}$. Moreover, part (i) of Lemma 1 implies that $c^{h\ell} = c^{\ell}$, and therefore $u'\left(c^{h\ell}\right) = u'\left(c^{\ell}\right)$. Under these circumstances, the difference between the marginal willingness to pay of a low-ability agent and of a mimicker is only related to the difference in their marginal (effort) cost of acquiring education.

[10] For example, assume that the function $\sigma(a)$ only applies to low-ability agents, whereas for high-ability agents (independently on whether they behave as mimickers or not) the cost-of-IM function is given by $k\sigma(a)$, with k representing a positive constant. We would have that $a^{h\ell} = a^{\ell} = 0$, and therefore $MRS^{h\ell}_{MB} = MRS^{\ell}_{MB}$, at those (e, M, B)-bundles satisfying the condition $w(e)\,u'(B) = v'(M/w(e))$; at all other (e, M, B)-bundles we would have that $a^{h\ell} \neq a^{\ell}$ and $MRS^{h\ell}_{MB} \neq MRS^{\ell}_{MB}$. Within a different setting, heterogeneous misreporting technologies are considered by Kopczuk (2001), Blomquist, Christiansen and Micheletto (2016), and Canta, Cremer and Gahvari (2021).

[11] The same would be true if one were to relax the assumption that the effort-cost of acquiring education and the effort cost of supplying labor in the market are additively separable in the agents' utility function. For example, assuming that $U = u(c) - g(v(L) + \varphi(\theta, e))$, with $g(\cdot)$ representing an increasing and convex function, would again imply that the results provided by parts (i) and (ii) of Lemma 1 break down. The intuition comes from observing that the reformulated utility function implies that the marginal disutility of labor supply is affected by the (effort) cost of acquiring education.

[12] On the other hand, if high-ability agents had access to a cheaper IM-technology than low-ability agents, it would still be true that the government would not benefit from the possibility to condition the tax liability on reported income.

gains arise from mimicking-deterring effects: requiring transfer-recipients to report a specific amount of income (on top of acquiring a given level of education) imposes a softer constraint on the labor supply of agents having access to a cheaper IM-technology. Therefore, if deserving agents have better IM-opportunities, requiring transfer-recipients to report a specific amount of income imposes a lighter burden on those who are the intended beneficiaries of the redistributive program than on mimickers.

Our final observation here is that the characterization of $T_M\left(M^\ell, e^\ell\right)$ and $T_e\left(M^\ell, e^\ell\right)$ provided by eqs. (19)-(20) are precisely the ones that emerge in a model without IM. This is quite intuitive in light of the fact that the no-IM case can be interpreted as a special case in which the condition $a^\ell = a^{h\ell}$ applies. Thus, if wage conditional on education and IM-costs are both type-independent, then IM will have no impact on the characterization of optimal marginal taxes on incomes and educational attainments.

3.4 Case (b): $\partial w\left(\theta, e\right)/\partial\theta > 0$

Lemma 2 is the counterpart of Lemma 1 for when $\partial w\left(\theta, e\right)/\partial\theta > 0$.

Lemma 2 *Assume that $\partial w\left(\theta, e\right)/\partial\theta > 0$. Then:*

(i) A high-ability worker, when behaving as a mimicker, will misreport more than a low-ability worker. That is, $a^{h\ell} > a^\ell$.

(ii) For any given (e, M, B)-bundle we have that $MRS^\ell_{MB} > MRS^{h\ell}_{MB}$.

Proof. See Appendix A. ■

The implication of Lemma 2 for the optimal tax characterization given in Proposition 1 is that $T_M\left(M^\ell, e^\ell\right) > 0$ which is the same result as the one in the absence of IM. Indeed, $T_M\left(M^\ell, e^\ell\right)$ has the same characterization as in the case without IM. Turning to $T_e\left(M^\ell, e^\ell\right)$, the interesting questions to ask is if $T_e\left(M^\ell, e^\ell\right)$ exceeds or falls short of p and what the role of IM in this is.

As a benchmark for our discussion, consider a setting where agents cannot misreport their earned income to the tax authority. In that case, we would have that $a^\ell = a^{h\ell} = 0$ and $c^\ell = c^{h\ell}$, so that $u'\left(c^\ell\right) = u'\left(c^{h\ell}\right)$. Then, eq. (17) simplifies to

$$\begin{aligned} T_e\left(M^\ell, e^\ell\right) &= p + \frac{\lambda u'\left(c^\ell\right)}{\mu\pi}\left[\epsilon^{h\ell}_{w,e} MRS^{h\ell}_{M,B} - \epsilon^{\ell}_{w,e} MRS^{\ell}_{M,B}\right] M^\ell \\ &\quad + \frac{\lambda}{\mu\pi}\left[\frac{\partial\varphi\left(\theta^\ell, e^\ell\right)}{\partial e^\ell} - \frac{\partial\varphi\left(\theta^h, e^\ell\right)}{\partial e^\ell}\right]. \end{aligned} \tag{21}$$

To make things simpler, let us make one further simplification and assume that the effort cost of achieving a given education level, depends only on the education level and not ability. That is, $\partial\varphi\left(\theta, e\right)/\partial\theta = 0$ so that $\partial\varphi\left(\theta^\ell, e^\ell\right)/\partial e^\ell = \partial\varphi\left(\theta^h, e^\ell\right)/\partial e^\ell \equiv \varphi'\left(e^\ell\right)$. Under this assumption, eq. (21) simplifies to

$$T_e\left(M^\ell, e^\ell\right) = p + \frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\epsilon^{h\ell}_{w,e} MRS^{h\ell}_{M,B} - \epsilon^{\ell}_{w,e} MRS^{\ell}_{M,B}\right] M^\ell. \tag{22}$$

We know that in that absence of IM, $MRS^{\ell}_{M,B} > MRS^{h\ell}_{M,B}$. Therefore, if $\epsilon^{h\ell}_{w,e} \leq \epsilon^{\ell}_{w,e}$, eq. (22) tells us that $T_e\left(M^\ell, e^\ell\right) < p$; this includes the case in which $w(\theta, e)$ can be written as $w(\theta, e) = f(\theta)\, g(e)$, where we have $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e} = g'(e)/g(e)$, and the one in which $w(\theta, e)$ can be written as $w(\theta, e) = f(\theta) + g(e)$, where we have (assuming $f' > 0$) $\epsilon^{h\ell}_{w,e} = \frac{g'(e)}{f(\theta^h)+g(e)} < \frac{g'(e)}{f(\theta^\ell)+g(e)} = \epsilon^{\ell}_{w,e}$. On the other hand, if $\epsilon^{h\ell}_{w,e} > \epsilon^{\ell}_{w,e}$, it is possible for $T_e\left(M^\ell, e^\ell\right)$ to exceed or fall short of p. What matters is the relative size of $\epsilon^{h\ell}_{w,e}/\epsilon^{\ell}_{w,e}$ as compared to $MRS^{\ell}_{M,B}/MRS^{h\ell}_{M,B}$. To interpret this result one should consider again whether or not, for a given (e, M, B)-bundle, a mimicker differs from a low-ability agent in terms of marginal willingness to pay for education (MWP_{ec}). If such a difference exists, then the government has an incentive, for mimicking-deterring purposes, to let $T_e\left(M^\ell, e^\ell\right)$ deviate from p. More specifically, if MWP_{ec} is higher (resp.: lower) for a mimicker than for a low-ability agent, it will be desirable to set $T_e\left(M^\ell, e^\ell\right)$ at a level that is higher (resp.: lower) than p.

Without IM, both agents have the same consumption and therefore the same marginal utility of consumption, $u'\left(c^{h\ell}\right) = u'\left(c^{\ell}\right) = u'\left(B^{\ell}\right)$. Thus, if they also incur the same marginal (effort) cost of acquiring education, a difference in their respective MWP_{ec} can only arise from a difference in how a marginal increase in education affects their disutility of labor supply. For a given (e, M, B)-bundle, this effect is given by

$$\frac{\partial v\left(M/w(\theta, e)\right)}{\partial e} = v'\left(\frac{M}{w(\theta, e)}\right)\frac{M}{\left(w(\theta, e)\right)^2}\frac{\partial w(\theta, e)}{\partial e} = M u'(B)\, MRS_{MB}\epsilon_{w,e}. \tag{23}$$

Hence, if $\epsilon^{h\ell}_{w,e} MRS^{h\ell}_{M,B} > \epsilon^{\ell}_{w,e} MRS^{\ell}_{M,B}$ a mimicker's marginal willingness to pay for education will exceed that of a low-ability agent, in which case it will be desirable to set $T_e\left(M^\ell, e^\ell\right) > p$, and vice versa for the case when $\epsilon^{h\ell}_{w,e} MRS^{h\ell}_{M,B} < \epsilon^{\ell}_{w,e} MRS^{\ell}_{M,B}$. This is indeed the message provided by eq. (22).

Consider now the counterpart of (22) in the presence of IM. Assuming again that $\partial\varphi(\theta, e)/\partial\theta = 0$, we have from eq. (17),

$$\begin{aligned} T_e\left(M^\ell, e^\ell\right) &= p + \frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[I^{h\ell}\epsilon^{h\ell}_{w,e} MRS^{h\ell}_{M,B} - I^{\ell}\epsilon^{\ell}_{w,e} MRS^{\ell}_{M,B}\right] \\ &\quad + \frac{\lambda}{\mu\pi}\frac{\varphi'\left(e^\ell\right)}{u'\left(c^\ell\right)}\left[u'\left(c^{h\ell}\right) - u'\left(c^{\ell}\right)\right]. \end{aligned} \tag{24}$$

Given that part (i) of Lemma 2 implies that $I^{h\ell} = M^\ell + a^{h\ell} > M^\ell + a^\ell = I^\ell$ and $c^{h\ell} = B^\ell + a^{h\ell} - \sigma\left(a^{h\ell}\right) > B^\ell + a^\ell - \sigma\left(a^\ell\right) = c^\ell$, by comparing (22) and (24) we can see that IM has an ambiguous effect on the sign of $T_e\left(M^\ell, e^\ell\right) - p$.[13]

[13] We must emphasize here that our discussion refers only to what the "tax rules" (22) and (24) suggest; it does not refer to the final equilibrium "levels" of the marginal tax. The common variables appearing in the tax

On one hand, given that $I^{h\ell} > I^{\ell}$, the second term on the RHS of (24) is lowered compared to the corresponding term in (22), and this makes it more likely that $T_e\left(M^{\ell}, e^{\ell}\right) > p$. To interpret this result, observe that a marginal increase in education, raising an agent's wage rate, allows earning a given amount of income at a lower labor supply, and that the magnitude of this labor-saving effect is increasing in the initial labor supply. With IM, the difference between the labor supply of a low-ability type and of a mimicker does not only depend on the difference in their wage rate but also on a difference in their earned income (for a given, common, reported income). The fact that the earned income of a mimicker exceeds that of a low-ability type tends to raise $MWP_{ec}^{h\ell}$ above MWP_{ec}^{ℓ}.

On the other hand, the RHS of (24) contains a third term which takes a negative sign (under the assumption that $u''(c) < 0$) and which was absent in (22). This term, which depends on the difference between the marginal utility of consumption of a high-ability mimicker and a low-ability type, works in the direction of favoring $T_e\left(M^{\ell}, e^{\ell}\right) < p$. To interpret this result, observe that the marginal willingness to pay for education is decreasing in the ratio $\frac{\partial\varphi(\theta,e)/\partial e}{u'}$. The fact that $c^{h\ell} > c^{\ell}$ implies that $u'\left(c^{h\ell}\right) < u'\left(c^{\ell}\right)$. Therefore, under the assumption that $\partial\varphi(\theta, e)/\partial\theta = 0$, the marginal effort cost of acquiring education lowers MWP_{ec}^{ℓ} to a smaller extent than $MWP_{ec}^{h\ell}$.

3.5 Income misreporting and distortion in educational attainment vis-à-vis labor supply

The discussion above centered around distortions in labor supply and educational attainment vis-à-vis consumption. An equally interesting perspective is to look at the distortion in educational attainment vis-à-vis labor supply. The condition for optimal educational attainment versus labor supply at a first-best allocation can be found through dividing laissez-faire condition (3) by laissez-faire condition (2). This results in

$$\frac{L}{w}\frac{\partial w}{\partial e} - \frac{\partial\varphi/\partial e}{v'} = \frac{p}{w}, \tag{25}$$

where the LHS of (25) shows the marginal rate of substitution between education and labor supply, MRS_{eL}.

In what follows we will rely on condition (25) to evaluate if, and how, an optimal tax policy distorts the educational attainment of an agent vis-à-vis his labor supply. More specifically, the education acquired by type-j agents (with $j = h, \ell$) will be downward (resp.: upward) distorted if it is the case that, at the solution to problem $\mathcal{P}1$, we have that $L^j \frac{\partial w\left(\theta^j, e^j\right)}{\partial e^j} - \frac{w\left(\theta^j, e^j\right)}{v'\left(L^j\right)} \frac{\partial\varphi\left(\theta^j, e^j\right)}{\partial e^j} - p >$ (resp.: $<$) 0.

formulas with and without IM assume different values in the two cases and do not allow a comparison between tax levels. The distinction between "tax rules" and "tax levels" was introduced by Atkinson and Stern (1974) in the context of the analysis of optimal provision of public goods in a first-best versus a second-best world.

Before proceeding further, it is worth noticing that when $T_M\left(M^j, e^j\right)$ (as defined by the RHS of (9)) is greater than zero, e^j could be downward distorted even when $T_e\left(M^j, e^j\right)$ (as defined by the RHS of (10)) is smaller than p. In particular, e^j will remain downward distorted as long as the inequality $T_e\left(M^j, e^j\right) > \left[1 - T_M\left(M^j, e^j\right)\right] p$ holds.[14]

Proposition 2 provides our main results regarding the optimal distortion on e^j vis-a-vis L^j.

Proposition 2 *Under an optimal nonlinear tax schedule $T(M, e)$:*

(i) The amount of education acquired by high-ability agents is undistorted vis-à-vis labor supply;

(ii) For the low-ability agents, e^ℓ is downward (resp.: upward) distorted vis-à-vis labor supply if the following condition holds:

$$MRS_{eM}^{h\ell} - MRS_{eM}^{\ell} > (\textit{resp.:}\ <)\, 0, \tag{26}$$

where, denoting $w\left(\theta^\ell, e^\ell\right)$ by w^ℓ and $w\left(\theta^h, e^\ell\right)$ by $w^{h\ell}$,

$$MRS_{eM}^{\ell} = I^\ell \epsilon_{w,e}^{\ell} - \frac{\partial \varphi\left(\theta^\ell, e^\ell\right)}{\partial e^\ell} \frac{w^\ell}{v'\left(L^\ell\right)}, \tag{27}$$

$$MRS_{eM}^{h\ell} = I^{h\ell} \epsilon_{w,e}^{h\ell} - \frac{\partial \varphi\left(\theta^h, e^\ell\right)}{\partial e^\ell} \frac{w^{h\ell}}{v'\left(L^{h\ell}\right)}. \tag{28}$$

Proof. See Appendix A. ■

The outcome in (i) is a direct consequence of the fact that there is no mimicking-deterring motive to distort the choices of high-ability agents. According to Proposition 1, at an optimum we have that $T_M\left(M^h, e^h\right) = 0$ and $T_e\left(M^h, e^h\right) = p$. Together, these two results trivially imply that $T_e\left(M^h, e^h\right) = \left[1 - T_M\left(M^h, e^h\right)\right] p$, i.e. that the educational attainment is undistorted for high-ability agents.

To get an intuition for the result stated in (ii), suppose to start from an initial situation where the (e, M, B)-bundle offered to low-ability agents satisfies the no-distortion condition $MRS_{eM}^{\ell} = p$.[15] If it is the case that $MRS_{eM}^{h\ell} > (<)\, MRS_{eM}^{\ell}$, introducing a small downward (upward) distortion on e^ℓ will entail a first-order utility loss on the high-ability mimicker, thereby

[14] The no-distortion condition $T_e\left(M^j, e^j\right) = \left[1 - T_M\left(M^j, e^j\right)\right] p$ can be easily interpreted when restated as

$$w\left(\theta^j, e^j\right)\left[1 - T_M\left(M^j, e^j\right)\right] / T_e\left(M^j, e^j\right) = w\left(\theta^j, e^j\right) / p.$$

Taking into account that $T_e\left(M^j, e^j\right)$ represents the implicit marginal (pecuniary) cost of education for an agent of ability j, the condition above states that, for a given ability-type and a given amount of education, the ratio between the net marginal return of labor supply and the marginal (pecuniary) cost of education should be the same as under laissez-faire. For example, under a linear income tax such a condition would be fulfilled when the pecuniary costs of education are fully deductible from the income tax base (since this implies that education is being subsidized at a rate that is equal to the marginal income-tax-rate).

[15] The marginal rate of substitution MRS_{eM}^{ℓ} provides a measure of the required variation in M^ℓ that would leave unchanged the utility of low-ability agents when e^ℓ is marginally increased. The marginal rate of substitution $MRS_{eM}^{h\ell}$ provides the corresponding amount for a high-ability mimicker.

relaxing the binding self-selection constraint, while having only second-order effects on the utility of low-ability agents and on the government's budget constraint. Clearly, a downward distortion is more likely to be desirable when the education elasticity of wage rate is increasing in ability and the marginal effort-cost of acquiring education is decreasing in ability (with the reverse directions for an upward distortion).

In order to evaluate the specific effects of IM, we again consider case (a) and case (b) separately.

3.6 Case (a): $\partial w(\theta, e)/\partial\theta = 0$

In Section 3.3 we have seen that, if wage conditional on education and IM-costs $\sigma(\cdot)$ are both type-independent, IM has no impact on the characterization of optimal marginal taxes on incomes and educational attainments. With or without IM we have that $T_M\left(M^\ell, e^\ell\right) = 0$ and $T_e\left(M^\ell, e^\ell\right) > p$ (under the assumption that $\partial^2\varphi(\theta, e)/\partial e\partial\theta < 0$). This also means that, with or without IM, the condition $T_e\left(M^\ell, e^\ell\right) > \left[1 - T_M\left(M^\ell, e^\ell\right)\right] p$ holds at an optimum, implying that e^ℓ is downward distorted. The same conclusion can be reached by evaluating the difference $MRS^{h\ell}_{eM} - MRS^{\ell}_{eM}$. Based on part (i) and (iii) of Lemma 1 we have that, with or without IM, $I^\ell \epsilon^\ell_{w,e} = I^{h\ell}\epsilon^{h\ell}_{w,e}$. Moreover, given that $w^\ell = w^{h\ell}$ and $L^\ell = L^{h\ell}$, the difference $MRS^{h\ell}_{eM} - MRS^{\ell}_{eM}$ boils down to

$$MRS^{h\ell}_{eM} - MRS^{\ell}_{eM} = \frac{w^\ell}{v'(L^\ell)}\left[\frac{\partial\varphi\left(\theta^\ell, e^\ell\right)}{\partial e^\ell} - \frac{\partial\varphi\left(\theta^h, e^\ell\right)}{\partial e^\ell}\right] > 0. \tag{29}$$

3.7 Case (b): $\partial w(\theta, e)/\partial\theta > 0$

To make matters simpler, assume also $\partial\varphi(\theta, e)/\partial\theta = 0$. Then, in the absence of IM, condition (26) would become

$$\begin{aligned}\left(\epsilon^{h\ell}_{w,e} - \epsilon^{\ell}_{w,e}\right) M^\ell + \left[\frac{w^\ell}{v'(L^\ell)} - \frac{w^{h\ell}}{v'(L^{h\ell})}\right]\frac{\partial\varphi\left(e^\ell\right)}{\partial e^\ell} \quad &> \quad 0 \text{ for a downward distortion,} \\ &< \quad 0 \text{ for an upward distortion.}\end{aligned} \tag{30}$$

Notice that the sign of the expression on the LHS of (30) is unambiguously negative when $\epsilon^{h\ell}_{w,e} \leq \epsilon^{\ell}_{w,e}$ (given that $w^\ell < w^{h\ell}$, $L^\ell = M^\ell/w^\ell > L^{h\ell} = M^\ell/w^{h\ell}$, and $v'' > 0$). Accordingly, education should be upward distorted for the low-ability agents even when $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e}$. This result is reminiscent of a similar one obtained by Bovenberg and Jacobs (2005) and is driven by the assumption that education entails both a resource- and an effort-cost.[16] As shown by

[16] See, in particular, eq. (42) on page 2022 of Bovenberg and Jacobs (2005). The result also hinges on the assumption that the effort-cost of acquiring education and the effort cost of supplying labor in the market are additively separable in the agents' utility function.

Maldonado (2008), in a setting without IM and where education entails only a pecuniary cost, e^ℓ should be left undistorted when $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e}$.[17]

With IM, condition (30) changes to

$$\left(I^{h\ell}\epsilon^{h\ell}_{w,e} - I^{\ell}\epsilon^{\ell}_{w,e}\right) + \left[\frac{w^\ell}{v'\left(L^\ell\right)} - \frac{w^{h\ell}}{v'\left(L^{h\ell}\right)}\right]\frac{\partial\varphi\left(e^\ell\right)}{\partial e^\ell} \begin{array}{l} > 0 \text{ for a downward distortion,} \\ < 0 \text{ for an upward distortion.} \end{array} \quad (31)$$

Comparing the first term on the LHS of (31) with the corresponding term in (30), we can see that the possibility of IM tends to favor distorting downwards e^ℓ.

Whereas in (30) the sign of the first term only depends on the difference $\epsilon^{h\ell}_{w,e} - \epsilon^{\ell}_{w,e}$ (since both a low-ability agent and a high-ability mimicker earn the same income), the fact that $I^{h\ell} = M^\ell + a^{h\ell} > M^\ell + a^\ell = I^\ell$ implies that the first term in (31) is still positive when $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e}$ and can remain positive also when $\epsilon^{h\ell}_{w,e} < \epsilon^{\ell}_{w,e}$.

With respect to the second term in (30) and (31), notice that they are formally identical. However, whereas in (30) we have that $L^\ell = M^\ell/w^\ell$ and $L^{h\ell} = M^\ell/w^{h\ell}$, which necessarily implies that $L^\ell > L^{h\ell}$, in (31) we have that $L^\ell = \left(M^\ell + a^\ell\right)/w^\ell$ and $L^{h\ell} = \left(M^\ell + a^{h\ell}\right)/w^{h\ell}$. Given that $a^{h\ell} > a^\ell$, with IM one can no longer be sure that $L^\ell > L^{h\ell}$. Notwithstanding this difference, the sign of the second term in (31) remains negative as it was the case for the second term in (30).[18]

3.8 An illustrative example

To elaborate more on the effects of IM, suppose that $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e} \equiv \epsilon_{w,e}$ and $v\left(L\right) = L^2/2$. The LHS of (31) can then be rewritten as

$$\left(I^{h\ell} - I^\ell\right)\epsilon_{w,e} + \left[\frac{\left(w^\ell\right)^2}{I^\ell} - \frac{\left(w^{h\ell}\right)^2}{I^{h\ell}}\right]\frac{\partial\varphi\left(e^\ell\right)}{\partial e^\ell}. \quad (32)$$

Furthermore, assume that preferences are quasi-linear in consumption, so that $u' = \varkappa > 0$ and $u'' = 0$, and that $\sigma\left(a\right) = ka^2/2$, where k is a positive constant. The latter assumption allows exploring how a change in the cost of IM, modeled as a variation in k, affects the two terms appearing in (32).

Start with the difference $I^{h\ell} - I^\ell = a^{h\ell} - a^\ell > 0$. With $u' = \varkappa$ we have that, for a given

[17] Maldonado (2008) also shows that e^ℓ should be downward distorted when $\epsilon^{h\ell}_{w,e} > \epsilon^{\ell}_{w,e}$ and upward distorted when $\epsilon^{h\ell}_{w,e} < \epsilon^{\ell}_{w,e}$. Noticing that $\partial\left(w^{h\ell}/w^\ell\right)/\partial e^\ell = \left(\epsilon^{h\ell}_{w,e} - \epsilon^{\ell}_{w,e}\right)w^{h\ell}/\left(e^\ell w^\ell\right)$, these results can be interpreted as stating that e^ℓ ought to be distorted only when the wage ratio $w^{h\ell}/w^\ell$ is a function of e^ℓ.

[18] To see this, notice that we can re-express the difference within square brackets in (31) as $\left[MRS^\ell_{MB}u'\left(c^\ell\right)\right]^{-1} - \left[MRS^{h\ell}_{MB}u'\left(c^{h\ell}\right)\right]^{-1}$. Given that from Lemma 2 we know that $MRS^\ell_{MB} > MRS^{h\ell}_{MB}$ and $c^\ell < c^{h\ell}$ (since $a^{h\ell} > a^\ell$), we can conclude that

$$\left[MRS^\ell_{MB}u'\left(c^\ell\right)\right]^{-1} - \left[MRS^{h\ell}_{MB}u'\left(c^{h\ell}\right)\right]^{-1} < 0,$$

and therefore the sign of the second term in (31) remains negative.

(e^ℓ, M^ℓ, B^ℓ)-bundle, the optimality condition for a is

$$(1-ka)\varkappa - \frac{M^\ell + a}{\left[w\left(\theta^j, e^\ell\right)\right]^2} = 0,$$

from which one obtains the explicit solution

$$a^j = \frac{\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa - M^\ell}{1+\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa k}. \tag{33}$$

Since we know that at an optimum $T_M\left(M^\ell, e^\ell\right) > 0$, we have that $a^\ell > 0$, and therefore $\left[w\left(\theta^\ell, e^\ell\right)\right]^2 \varkappa - M^\ell > 0$. Moreover, since we know that $a^{h\ell} > a^\ell$, we also have that $\left[w\left(\theta^h, e^\ell\right)\right]^2 \varkappa - M^\ell > 0$.[19]

Taking into account that, from eq. (33), we have

$$\frac{\partial a^j}{\partial k} = -\frac{\left\{\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa - M^\ell\right\}\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa}{\left\{1+\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa k\right\}^2} = -a^j \frac{\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa}{1+\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa k} < 0, \tag{34}$$

it follows that the derivative $\partial\left(I^{h\ell} - I^\ell\right)/\partial k$ is given by

$$\frac{\partial\left(I^{h\ell} - I^\ell\right)}{\partial k} = \frac{\partial\left(a^{h\ell} - a^\ell\right)}{\partial k} = \left\{a^\ell \frac{\left[w\left(\theta^\ell, e^\ell\right)\right]^2}{1+\left[w\left(\theta^\ell, e^\ell\right)\right]^2 \varkappa k} - a^{h\ell} \frac{\left[w\left(\theta^h, e^\ell\right)\right]^2}{1+\left[w\left(\theta^h, e^\ell\right)\right]^2 \varkappa k}\right\}\varkappa < 0, \tag{35}$$

where the sign of the inequality follows from the fact that $a^\ell < a^{h\ell}$ and the ratio $\frac{\left[w\left(\theta, e^\ell\right)\right]^2}{1+\left[w\left(\theta, e^\ell\right)\right]^2 \varkappa k}$ is increasing in θ. Thus, according to eq. (35), the difference $I^{h\ell} - I^\ell$ gets larger as IM becomes easier (i.e., k goes down).

Now consider the difference $\left(w^\ell\right)^2/I^\ell - \left(w^{h\ell}\right)^2/I^{h\ell} < 0$ appearing in (32).[20] For a given (e^ℓ, M^ℓ, B^ℓ)-bundle we have that

$$\frac{\partial\left(\frac{\left(w^\ell\right)^2}{I^\ell} - \frac{\left(w^{h\ell}\right)^2}{I^{h\ell}}\right)}{\partial k} = \frac{\left[w\left(\theta^h, e^\ell\right)\right]^2 \frac{\partial a^{h\ell}}{\partial k}}{\left(M^\ell + a^{h\ell}\right)^2} - \frac{\left[w\left(\theta^\ell, e^\ell\right)\right]^2 \frac{\partial a^\ell}{\partial k}}{\left(M^\ell + a^\ell\right)^2} = \frac{\frac{\partial a^{h\ell}}{\partial k}}{\left(L^{h\ell}\right)^2} - \frac{\frac{\partial a^\ell}{\partial k}}{\left(L^\ell\right)^2}. \tag{36}$$

Noticing that from (33) we have that

$$L^{h\ell} = \frac{M^\ell + a^{h\ell}}{w\left(\theta^h, e^\ell\right)} = \frac{\left(M^\ell k + 1\right) w\left(\theta^h, e^\ell\right)\varkappa}{1+\left[w\left(\theta^h, e^\ell\right)\right]^2 \varkappa k}, \tag{37}$$

[19] Notice that, with $u'' = 0$ and $u' = \varkappa$, an agent of ability j would choose to earn $\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa$ when the tax liability is only conditioned on the amount of acquired education and he is forced to acquire an amount of education e^ℓ. When the tax is conditioned on both education and reported income, agents exploit IM to adjust their labor supply and fill part of the gap between $\left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa$ and M^ℓ. As shown by eq. (33), how much of this gap is optimally filled depends on k. At the limit, if IM were costless, we have from eq. (33) that $\lim_{k \to 0^+} a^j = \left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa - M^\ell$, in which case $I^j = \left[w\left(\theta^j, e^\ell\right)\right]^2 \varkappa$, i.e., $L^j = \varkappa w\left(\theta^j, e^\ell\right)$.

[20] With $v(L) = L^2/2$ and $u' = \varkappa$, we have that $MRS^\ell_{MB} = L^\ell/\left(\varkappa w^\ell\right) = I^\ell/\left[\left(w^\ell\right)^2 \varkappa\right]$ and $MRS^{h\ell}_{MB} = L^{h\ell}/\left(\varkappa w^{h\ell}\right) = I^{h\ell}/\left[\left(w^{h\ell}\right)^2 \varkappa\right]$. Therefore, the difference $\left(w^\ell\right)^2/I^\ell - \left(w^{h\ell}\right)^2/I^{h\ell}$ can be re-expressed as $\left[\left(MRS^\ell_{MB}\right)^{-1} - \left(MRS^\ell_{MB}\right)^{-1}\right]/\varkappa$, which is negative given that $MRS^\ell_{MB} - MRS^{h\ell}_{MB} > 0$ according to part (ii) of Lemma 2.

and

$$L^{\ell}=\frac{M^{\ell}+a^{\ell}}{w\left(\theta^{\ell},e^{\ell}\right)}=\frac{\left(M^{\ell}k+1\right)w\left(\theta^{\ell},e^{\ell}\right)\varkappa}{1+\left[w\left(\theta^{\ell},e^{\ell}\right)\right]^{2}\varkappa k},\tag{38}$$

substituting (37)-(38) in (36), and using (34), gives

$$\begin{aligned}\frac{\frac{\partial a^{h\ell}}{\partial k}}{\left(L^{h\ell}\right)^{2}}-\frac{\frac{\partial a^{\ell}}{\partial k}}{\left(L^{\ell}\right)^{2}} &= -\frac{\left\{\left[w\left(\theta^{h},e\right)\right]^{2}\varkappa-M\right\}\left[w\left(\theta^{h},e\right)\right]^{2}\varkappa}{\left\{1+\left[w\left(\theta^{h},e\right)\right]^{2}\varkappa k\right\}^{2}}\frac{\left\{1+\left[w\left(\theta^{h},e\right)\right]^{2}\varkappa k\right\}^{2}}{\left[\left(1+Mk\right)w\left(\theta^{h},e\right)\varkappa\right]^{2}}\\ &\quad+\frac{\left\{\left[w\left(\theta^{\ell},e\right)\right]^{2}\varkappa-M\right\}\left[w\left(\theta^{\ell},e\right)\right]^{2}\varkappa}{\left\{1+\left[w\left(\theta^{\ell},e\right)\right]^{2}\varkappa k\right\}^{2}}\frac{\left\{1+\left[w\left(\theta^{\ell},e\right)\right]^{2}\varkappa k\right\}^{2}}{\left[\left(1+Mk\right)w\left(\theta^{\ell},e\right)\varkappa\right]^{2}}\\ &= \frac{\left[w\left(\theta^{\ell},e\right)\right]^{2}-\left[w\left(\theta^{h},e\right)\right]^{2}}{\left(1+Mk\right)^{2}}<0,\end{aligned}$$

implying that, as IM becomes easier (k goes down), the difference $\left(w^{\ell}\right)^{2}/I^{\ell}-\left(w^{h\ell}\right)^{2}/I^{h\ell}$ becomes smaller in absolute value.[21]

We can then conclude that, as it becomes easier for agents to engage in IM, the first term in (32) tends to get larger and the second term tends to become smaller in absolute value. Both of these effects strengthen the case for downward distorting e^{ℓ}.

4 IM and linear consumption taxes

We have thus far neglected the possibility of using a consumption tax as an additional policy instrument. In a setting without IM, a consumption tax would be redundant: with $c^{h\ell}=c^{\ell}=B^{\ell}$ the introduction of a consumption tax (subsidy) would hurt (benefit) a high-ability mimicker in the same way as it would hurt (benefit) a true low-ability type.[22] With IM, however, a consumption tax ceases to be redundant given the possibility that $c^{\ell}\neq c^{h\ell}$. For this reason, we will now evaluate how our previous qualitative results are affected if the government supplements an optimal nonlinear tax with a consumption tax.

Denote the consumption tax rate by t so that the consumer price of c is $1+t$.[23] Adding t as

[21] At the limit, when k approaches zero, the difference $\left(w^{\ell}\right)^{2}/I^{\ell}-\left(w^{h\ell}\right)^{2}/I^{h\ell}$ approaches zero given that both terms tend to $1/\varkappa$.

[22] More generally, relaxing our assumption of a single consumption good, we also know from the Atkinson-Stiglitz (1976) theorem that (uniform or non-uniform) commodity taxation would be a redundant instrument, in the absence of IM, as long as labor supply is weakly separable from the vector of consumption goods in the individuals' utility function.

[23] Given that we have normalized to 1 the producer price of c, t can equivalently be interpreted as a *specific* or an *ad valorem* tax rate. Notice also that, since the consumer price of c is $1+t$, it must be that $t>-1$. The assumption of linear taxation is made for realism. It is justified by the idea that the tax administration has information on anonymous transactions but not on the identity of the consumers. That is, the administration does not observe who bought how much; it only observes the total sales of a commodity. While this is a common approach in the optimal tax literature, it leaves aside the possibility that also consumption taxation is vulnerable to evasion. Within an optimal tax framework, the consequences of commodity tax evasion have been investigated by, among others, Usher (1986), Kaplow (1990) and Cremer and Gahvari (1993).

an additional policy instrument, the government's problem (hereafter, problem $\mathcal{P}2$) becomes

$$\max_{e^\ell, M^\ell, B^\ell, e^h, M^h, B^h, t} \quad u\left(\frac{B^\ell + a^\ell - \sigma\left(a^\ell\right)}{1+t}\right) - v\left(\frac{M^\ell + a^\ell}{w\left(\theta^\ell, e^\ell\right)}\right) - \varphi\left(\theta^\ell, e^\ell\right)$$

subject to

$$u\left(\frac{B^h + a^h - \sigma\left(a^h\right)}{1+t}\right) - v\left(\frac{M^h + a^h}{w\left(\theta^h, e^h\right)}\right) - \varphi\left(\theta^h, e^h\right) \geq \overline{V},$$

$$\begin{aligned} & u\left(\frac{B^h + a^h - \sigma\left(a^h\right)}{1+t}\right) - v\left(\frac{M^h + a^h}{w\left(\theta^h, e^h\right)}\right) - \varphi\left(\theta^h, e^h\right) \\ \geq \; & u\left(\frac{B^\ell + a^{h\ell} - \sigma\left(a^{h\ell}\right)}{1+t}\right) - v\left(\frac{M^\ell + a^{h\ell}}{w\left(\theta^h, e^\ell\right)}\right) - \varphi\left(\theta^h, e^\ell\right), \end{aligned}$$

$$\begin{aligned} & \pi\left[M^\ell - B^\ell + \frac{t}{1+t}\left(B^\ell + a^\ell - \sigma\left(a^\ell\right)\right) - pe^\ell\right] \\ & + (1-\pi)\left[M^h - B^h + \frac{t}{1+t}\left(B^h + a^h - \sigma\left(a^h\right)\right) - pe^h\right] \geq \overline{R}. \end{aligned}$$

Denote the Lagrange multipliers associated with the first, second, and third constraint by δ, λ and μ. The optimal values of $e^h, M^h, B^h, e^\ell, M^\ell, B^\ell, t$ are determined by the first-order conditions to this problem presented in Appendix B [equations (B1)–(B7)]. There, we prove the following proposition.

Proposition 3 *In a setting where the government supplements the optimal nonlinear tax $T(M, e)$ with a linear consumption tax, the consumption tax rate t is*

(i) a redundant instrument if $\partial w(\theta, e)/\partial\theta = 0$.

(ii) strictly positive if $\partial w(\theta, e)/\partial\theta > 0$.

Proof. See Appendix B. ■

The results follow from the characterization of the optimal value of t that we provide in the proof of Proposition 3. There, we show that

$$t = \frac{\lambda u'\left(c^{h\ell}\right)}{\mu} \frac{c^\ell - c^{h\ell}}{\pi\left[1 - \sigma'\left(a^\ell\right)\right]\left(\frac{da^\ell}{dt} + \frac{da^\ell}{dB^\ell}c^\ell\right) + (1-\pi)\left[1 - \sigma'\left(a^h\right)\right]\left(\frac{da^h}{dt} + \frac{da^h}{dB^h}c^h\right)}, \tag{39}$$

and we also prove that the denominator of the expression for t is negative, so that $sign(t) = sign\left(c^{h\ell} - c^\ell\right)$, and that Lemmas 1 and 2 remain valid when the set of policy instruments includes t. With $c^{h\ell} = c^\ell$ when $\partial w(\theta, e)/\partial\theta = 0$, eq. (39) implies that $t = 0$: consumption taxation is a redundant instrument. On the other hand, when $\partial w(\theta, e)/\partial\theta > 0$, $c^{h\ell} > c^\ell$ and $t > 0$. Contrary to the traditional result when there is no IM, optimal general income-education taxation benefits from being supplemented by consumption taxation.

That $t = 0$ when $\partial w(\theta, e)/\partial\theta = 0$ and $t > 0$ when $\partial w(\theta, e)/\partial\theta > 0$ comes from the effect of t on the self-selection constraint faced by the government in designing the nonlinear tax $T(M, e)$. Whereas in the former case, the introduction of t does not relax this self-selection constraint, it does so in the latter case. To see this, consider the following perturbation. Starting from a pre-reform equilibrium where $t = 0$, raise t by $dt > 0$ while at the same time adjust B^j (for $j = \ell, h$) by $dB^j = c^j dt$. Observe first that, by construction, the reform is welfare-neutral for low-skilled agents and for high-skilled agents (when not behaving as mimickers).[24]

Second, the reform has no effect on the government's budget. On the one hand, the upward adjustment of B^j changes tax revenue by

$$\begin{aligned}&-\pi dB^\ell - (1-\pi)\, dB^h \\ &+\pi\frac{t}{1+t}\left\{1+\left[1-\sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dB^\ell}\right\}dB^\ell + (1-\pi)\frac{t}{1+t}\left\{1+\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dB^h}\right\}dB^h,\end{aligned}$$

which is equal to $-\left[\pi c^\ell + (1-\pi)\, c^h\right] dt$ at the pre-reform equilibrium where $t = 0$, given that $dB^j = c^j dt$. On the other hand, the increase in t changes revenue by

$$\pi\frac{1}{1+t}\left\{c^\ell + t\left[1-\sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dt}\right\}dt + (1-\pi)\frac{1}{1+t}\left\{c^h + t\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dt}\right\}dt, \tag{40}$$

which is equal to $\left[\pi c^\ell + (1-\pi)\, c^h\right] dt$ at the pre-reform equilibrium where $t = 0$.

Third, what is left to consider is the effect of the reform on the utility of a high-skilled agent behaving as a mimicker. This effect is given by

$$\begin{aligned}dU^{h\ell} = \frac{dU^{h\ell}}{dB^\ell}dB^\ell + \frac{dU^{h\ell}}{dt}dt = \left(\frac{dU^{h\ell}}{dB^\ell}c^\ell + \frac{dU^{h\ell}}{dt}\right)dt = \\ \left(\frac{u'\left(c^{h\ell}\right)}{1+t}c^\ell - \frac{u'\left(c^{h\ell}\right)}{1+t}c^{h\ell}\right)dt = \frac{u'\left(c^{h\ell}\right)}{1+t}\left(c^\ell - c^{h\ell}\right)dt,\end{aligned}$$

which is equal to $\left(c^\ell - c^{h\ell}\right) u'\left(c^{h\ell}\right) dt$ at the pre-reform equilibrium where $t = 0$. Thus, if $c^\ell - c^{h\ell} = 0 \Rightarrow dU^{h\ell} = 0$. On the other hand, $c^\ell - c^{h\ell} < 0 \Rightarrow dU^{h\ell} < 0$ so that the reform has a detrimental effect on a mimicker's utility. This eases the self-selection constraint faced by the government in the design of the redistributive tax policy.

The expression for t in eq. (39) can thus be interpreted as providing the value of t that strikes an optimal balance between the mimicking-deterring effects of a marginal *compensated* increase in t (i.e., a marginal increase in t which is accompanied by adjusting upwards B^j, for $j = \ell, h$, by $dB^j = c^j dt$) and the revenue losses due to the behavioral responses on a^j (which shrink the aggregate consumption-tax base).

4.1 Consumption taxation and the properties of $T(M, e)$

Having shown that IM creates a role for consumption taxation when $\partial w(\theta, e)/\partial\theta > 0$, it will be interesting to evaluate how such a tax affects the features of the accompanying tax $T(M, e)$.

[24] Given that $dU^j/dB^j = u'\left(c^j\right)/(1+t)$ and $dU^j/dt = -u'\left(c^j\right)c^j/(1+t)$, we have that $dU^j = 0$ when $dB^j = c^j dt$.

For space considerations, we will concentrate only on the subject of optimal marginal income taxes which is widely studied in the literature. Nevertheless, we will later also discuss the effects of consumption taxation on the distortion of educational attainment vis-à-vis labor supply. Proposition 4 below presents our results relating to the optimal marginal income tax rates.

Proposition 4 *Assume that $\partial w(\theta, e)/\partial\theta > 0$ and that a nonlinear tax $T(M, e)$ is supplemented by a consumption tax levied at rate t. At an optimum, the implicit marginal tax rates on reported incomes are given by:*

$$T_M\left(M^h, e^h\right) = -\frac{t}{1+t}\left\{MRS^h_{MB} + \left[1-\sigma'\left(a^h\right)\right]\left(\frac{da^h}{dM^h} + MRS^h_{MB}\frac{da^h}{dB^h}\right)\right\} < 0, \quad (41)$$

$$\begin{aligned} T_M\left(M^\ell, e^\ell\right) &= \frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\frac{1}{1+t}\left(MRS^\ell_{MB} - MRS^{h\ell}_{MB}\right) \\ &\quad -\frac{t}{1+t}\left\{MRS^\ell_{MB} + \left[1-\sigma'\left(a^\ell\right)\right]\left(\frac{da^\ell}{dM^\ell} + MRS^\ell_{MB}\frac{da^\ell}{dB^\ell}\right)\right\}, \end{aligned} \quad (42)$$

where

$$MRS^j_{MB} + \left[1-\sigma'\left(a^j\right)\right]\left(\frac{da^j}{dM^j} + MRS^j_{MB}\frac{da^j}{dB^j}\right) > 0, \quad \textit{for} \;\; j = h, \ell. \quad (43)$$

Proof. See Appendix B. ■

To interpret eq. (41), recall that there are no mimicking-deterring reasons to distort the bundle offered to high-ability agents. Therefore, for a given value of t, the bundle (e^h, M^h, B^h) should be chosen in such a way that it maximizes the revenue collected from high-ability agents while at the same time allowing them to achieve the utility target $\overline{V}$ set in problem $\mathcal{P}2$. This is precisely the message provided by (41). To see this, suppose to start from a supposedly optimal equilibrium where high-ability agents are offered the bundle (e^h, M^h, B^h), and consider the effects of a small perturbation that raises M^h and B^h by, respectively, dM^h and $dB^h = MRS^h_{MB}dM^h$. The reform has no impact on the binding self-selection constraint: for high-ability agents not behaving as mimickers the reform is welfare neutral by construction; for high-ability mimickers the reform is welfare-neutral because the (e^ℓ, M^ℓ, B^ℓ)-bundle did not change (and for the same reason it has no impact on low-ability agents). What the reform does is to change the high-ability agent's consumption level, $c^h = \left(B^h + a^h - \sigma\left(a^h\right)\right)/(1+t)$, and the tax revenue collected from him, $M^h - B^h + \left(B^h + a^h - \sigma\left(a^h\right)\right)t/(1+t)$. Consumption changes by dc^h and consumption-tax revenue by tdc^h, where

$$dc^h = \frac{MRS^h_{MB} + \left[1-\sigma'\left(a^h\right)\right]\left(\frac{da^h}{dM^h} + MRS^h_{MB}\frac{da^h}{dB^h}\right)}{1+t}dM^h, \quad (44)$$

with the first term on the RHS of (44) capturing the direct effect of an increase in B^h and the second term capturing the behavioral effect of the reform, working through a change in a^h.

Therefore, denoting by ΔR the total effect of the reform on government's revenue, we have that

$$\Delta R = \left\{ 1 - MRS_{MB}^{h} + t \frac{MRS_{MB}^{h} + \left[1 - \sigma'\left(a^{h}\right)\right]\left(\frac{da^{h}}{dM^{h}} + MRS_{MB}^{h}\frac{da^{h}}{dB^{h}}\right)}{1+t} \right\} dM^{h}. \quad (45)$$

Given that condition (11) also applies to a setting where the tax function $T(M, e)$ is supplemented with a linear consumption tax (see Appendix B), eq. (45) can be equivalently restated as

$$\Delta R = \left\{ T_{M}\left(M^{h}, e^{h}\right) + t \frac{MRS_{MB}^{h} + \left[1 - \sigma'\left(a^{h}\right)\right]\left(\frac{da^{h}}{dM^{h}} + MRS_{MB}^{h}\frac{da^{h}}{dB^{h}}\right)}{1+t} \right\} dM^{h}. \quad (46)$$

If it turns out that $\Delta R \neq 0$, we would necessarily have to conclude that the initial (e^{h}, M^{h}, B^{h})-bundle had been chosen suboptimally. If $\Delta R > 0$ the suggested reform would allow the government to raise more revenue without prejudice for the utility of low- and high-ability agents, and without violating the self-selection constraint. If $\Delta R < 0$ the same outcome would be achieved by changing the direction of the reform ($dM^{h} < 0$). Thus, eq. (41) tells us that, if the (e^{h}, M^{h}, B^{h})-bundle is chosen optimally (for given t), a small perturbation of the kind that we have considered would leave unaffected the total revenue collected from high-ability agents.

Furthermore, notice that the perturbation that we have analyzed leads to an increase in the consumption-tax revenue collected from high-ability agents. This is because $t > 0$ at an optimum and since (43) ensures that dc^{h}, as given by (44), is greater than zero. Thus, in order to get $\Delta R = 0$ in (46), it must be that $T_{M}\left(M^{h}, e^{h}\right) < 0$: high-ability agents should face a negative marginal tax rate on reported income. This is meant to let them fully internalize the positive fiscal externality (on consumption-tax revenue) stemming from a marginal increase in M^{h}. It is worth noticing, however, that IM dampens the magnitude of the positive fiscal externality that we have discussed. This is due to the fact that, as we show in the Appendix B, the term $\left[1 - \sigma'\left(a^{h}\right)\right]\left(\frac{da^{h}}{dM^{h}} + MRS_{MB}^{h}\frac{da^{h}}{dB^{h}}\right)$ in (44) is negative. Recall that this term captures the behavioral response, through an adjustment in a^{h}, to the reform. Hence, while the envisaged reform has an overall positive effect on the revenue collected from high-ability agents through consumption taxation, the response along the IM-margin dampens this effect.

Turning to $T_{M}\left(M^{\ell}, e^{\ell}\right)$, the second term on the RHS of (42) has the same structure (and sign) as the term appearing on the RHS of (41), and it admits a similar interpretation. It represents the opposite of the increase in the consumption-tax revenue that would be collected from low-ability agents if they were induced to marginally increase their reported income along an indifference curve in the (M, B)-space (for given e^{ℓ}).[25] Given that, from Lemma 2, the first term

[25] Also in this case, albeit the sum $MRS_{MB}^{\ell} + \left[1 - \sigma'\left(a^{\ell}\right)\right]\left(\frac{da^{\ell}}{dM^{\ell}} + MRS_{MB}^{\ell}\frac{da^{\ell}}{dB^{\ell}}\right)$ is greater than zero, the component $\left[1 - \sigma'\left(a^{\ell}\right)\right]\left(\frac{da^{\ell}}{dM^{\ell}} + MRS_{MB}^{\ell}\frac{da^{\ell}}{dB^{\ell}}\right)$ is smaller than zero. Hence, the increase in consumption-tax-revenue is dampened by the behavioral response along the IM-margin.

on the RHS of (42) is positive, the sign of $T_M\left(M^\ell, e^\ell\right)$ remains ambiguous. Nevertheless, the takeaway message from (42) is that fiscal externality considerations warrant to lower $T_M\left(M^\ell, e^\ell\right)$ below the value that would be optimal based solely on self-selection considerations.

4.2 Consumption taxation and distortion in educational attainment

We now turn attention to how consumption taxation affects the optimal distortions on e^ℓ and e^h. Recall from Section 3.5 that, for any given type of agent, education is distorted with respect to labor supply when $L^j\frac{\partial w^j}{\partial e^j} - \frac{w^j}{v'(L^j)}\frac{\partial\varphi(\theta^j, e^j)}{\partial e^j} - p \neq 0$. The next proposition summarizes the main results about the optimal distortions on e^j.

Proposition 5 *Assume that $\partial w(\theta, e)/\partial\theta > 0$ and that a nonlinear tax $T(M, e)$ is supplemented by a consumption tax levied at rate t. At an optimum:*

(i) The amount of education acquired by high-ability agents is distorted upwards. In particular, we have that

$$L^h\frac{\partial w^h}{\partial e^h} - \frac{w^h}{v'(L^h)}\frac{\partial\varphi\left(\theta^h, e^h\right)}{\partial e^h} - p \quad = \quad -\frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\left(\frac{da^h}{de^h} + MRS^h_{eM}\frac{da^h}{dM^h}\right) < 0. \tag{47}$$

(ii) For low-ability agents, the distortion on e^ℓ has, in general, an ambiguous direction. In particular, we have that

$$\begin{aligned}&L^\ell\frac{\partial w^\ell}{\partial e^\ell} - \frac{w^\ell}{v'(L^\ell)}\frac{\partial\varphi\left(\theta^\ell, e^\ell\right)}{\partial e^\ell} - p = \\ &\frac{\lambda}{\mu\pi}\frac{v'\left(L^{h\ell}\right)}{w^{h\ell}}\left(MRS^{h\ell}_{eM} - MRS^\ell_{eM}\right) - \frac{t}{1+t}\left[1-\sigma'\left(a^\ell\right)\right]\left(\frac{da^\ell}{de^\ell} + MRS^\ell_{eM}\frac{da^\ell}{dM^\ell}\right).\end{aligned} \tag{48}$$

(iii) For both types,

$$\frac{da^j}{de^j} + MRS^j_{eM}\frac{da^j}{dM^j} > 0,\ j = h, \ell. \tag{49}$$

Proof. See Appendix B. ■

To interpret eq. (47) we can rely on an approach that is similar to the one used in Section 4.1. Suppose the government changes the high-ability agents' initial (e^h, M^h, B^h)-bundle via increasing e^h by de^h while simultaneously adjusting M^h in a manner that keeps their utility unaffected. This can be achieved by setting $dM^h = MRS^h_{eM}de^h$ (while keeping B^h at its initial value). Notice that the reform has no impact on the binding self-selection constraint: for high-ability agents not behaving as mimickers the reform is welfare neutral by construction; for high-ability mimickers the reform is welfare-neutral because the (e^ℓ, M^ℓ, B^ℓ)-bundle did not change (and for the same reason it has no impact on low-ability agents). What the reform does is to induce the high-ability agents to increase their a^h as seen from (49), which in turn increases their consumption $c^h = \left(B^h + a^h - \sigma\left(a^h\right)\right)/(1+t)$ by dc^h, where

$$dc^h = \frac{\left[1-\sigma'\left(a^h\right)\right]\left(\frac{da^h}{de^h} + MRS^h_{eM}\frac{da^h}{dM^h}\right)}{1+t}de^h > 0, \tag{50}$$

with the RHS of (50) capturing the behavioral effect of the reform, which is due to a change in the optimal amount of IM.

Therefore, denoting by ΔR the total effect of the reform on government's revenue, we have

$$\Delta R = \left\{MRS^h_{eM} - p + t\frac{\left[1-\sigma'\left(a^h\right)\right]\left(\frac{da^h}{de^h} + MRS^h_{eM}\frac{da^h}{dM^h}\right)}{1+t}\right\}de^h, \tag{51}$$

or equivalently, given that $MRS^h_{eM} = L^h\frac{\partial w^h}{\partial e^h} - \frac{w^h}{v'\left(L^h\right)}\frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h}$,

$$\Delta R = \left\{L^h\frac{\partial w^h}{\partial e^h} - \frac{w^h}{v'\left(L^h\right)}\frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h} - p + t\frac{\left[1-\sigma'\left(a^h\right)\right]\left(\frac{da^h}{de^h} + MRS^h_{eM}\frac{da^h}{dM^h}\right)}{1+t}\right\}de^h. \tag{52}$$

Observe that the proposed reform has a positive impact on the revenue collected from consumption taxation. This is because $t > 0$ at an optimum, and since dc^h, as given by (50), is greater than zero.[26] Notice also that, if education was undistorted (vis-à-vis labor supply) at the initial (e^h, M^h, B^h)-bundle, the extra cost for education (pde^h), paid for by the government, would be exactly matched by the increase in revenue (dM^h) collected through the nonlinear tax $T(M,e)$. Thus, leaving education undistorted for high-ability agents would not be revenue-maximizing for the government. The result stated in (47) can then be interpreted in Pigouvian terms: an upward distortion on e^h is justified as a way to internalize the fiscal externality (on consumption-tax-revenue) associated with the type of reform that we have described.

Consider next eq. (48) pertaining to low-ability agents. The first term on its RHS reflects mimicking-deterring considerations. Its sign coincides with that of $MRS^{h\ell}_{eM} - MRS^{\ell}_{eM}$. In the absence of consumption taxation, the sign of this difference is the sole determinant of the direction of the optimal distortion on e^ℓ (see part (ii) of Proposition 2): when $MRS^{h\ell}_{eM} - MRS^{\ell}_{eM} > 0$, e^ℓ should be downward distorted and when $MRS^{h\ell}_{eM} - MRS^{\ell}_{eM} < 0$, e^ℓ should be upward distorted.

The effect of consumption taxation manifests itself in the second term on the RHS of (48). This term, which favors an upward distortion on e^ℓ, captures fiscal-externality considerations of the same kind that we have previously discussed when analyzing eq. (47). It represents

[26] In the Appendix B we show that condition (11) applies also to a setting where the tax function $T(M,e)$ is supplemented with a linear consumption tax. Therefore, from (41) we can conclude that $1-\sigma'\left(a^h\right) > 0$. Exploiting the results stated by (49), and since $t > 0$, it then follows that dc^h, as given by the RHS of (50), is positive.

the opposite of the increase in the consumption-tax revenue that would be collected from low-ability agents if they were induced to marginally increase e^{ℓ} along an indifference curve in the (e, M)-space (for given B^{ℓ}). Hence, whereas mimicking-deterring considerations warrant a downward distortion on e^{ℓ}, the fiscal-externality considerations exert a countervailing effect. On the other hand, when mimicking-deterring considerations warrant an upward distortion on e^{ℓ}, the distortion is further magnified by the fiscal-externality considerations associated with consumption taxation.

5 An alternative specification of the wage function

In our analysis we have assumed that e, the amount of education acquired by an agent, is at the same time the sole endogenous variable that determines both an individual's labor productivity (through the function $w(\theta, e)$) and his/her effort cost (through the function $\varphi(\theta, e)$). Alternatively, one could have explicitly distinguished between, on one hand, the level of education achieved by an agent and, on the other, his/her learning effort in acquiring education. For instance, denoting by e the level of education and by z the learning effort exerted by an agent, we could have assumed that under laissez-faire agents choose e, z and L to maximize

$$U = u\left(w\left(\theta, e, z\right) L - pe\right) - v\left(L\right) - \varphi\left(\theta, z\right), \tag{53}$$

with the function $w(\theta, e, z)$ possessing the following properties:

$$\frac{\partial w(\theta, e, z)}{\partial \theta} \geq 0, \frac{\partial w(\theta, e, z)}{\partial e} > 0 \quad \text{and} \quad \frac{\partial w(\theta, e, z)}{\partial z} > 0.$$

Under this formulation, a realistic informational assumption would be that e, but not z, is observable by the government at the individual level; put differently, educational investments are publicly observable only partially and the government is restricted to a nonlinear function $T(M, e)$.[27] Notice that in this case, by virtue of the envelope theorem (for any given (e, M, B)-bundle agents always choose optimally z), the relevant condition to assess whether e is distorted (with respect to labor supply) becomes $L^j \frac{\partial w\left(\theta^j, e^j, z^j\right)}{\partial e^j} - p \neq 0$ or, equivalently, $\left(I^j \epsilon^j_{w,e}/e^j\right) - p \neq 0$.

As we show in the Appendix C, the model that we have considered in the previous sections can be reinterpreted as implicitly assuming that e and z are perfect complements. However, it is worth pointing out that relaxing this assumption and allowing for some degree of substitutability between e and z would not affect our qualitative results. If anything, it would strengthen them, especially the result that IM favors downward distorting e^{ℓ}.

To illustrate this point, we will here consider the case when ability only affects an agent's effort cost of acquiring education (i.e., $\partial w/\partial \theta = 0$ and $\partial \varphi/\partial \theta < 0$). Remember that in Section

[27] See, e.g., Kapicka and Neira (2019) for a model where human capital investments are only partially observable by the planner.

3 we have shown that, when $w = w(e)$, IM has no bite on the direction of the optimal distortion on e^ℓ. With or without IM, a nonlinear tax on education is the only instrument needed by the government and e^ℓ should be downward distorted. Assuming instead that $w = w(e, z)$, we get that, for a given (e, M, B)-bundle, $z^{h\ell} > z^\ell$;[28] therefore, despite the fact that by assumption the wage function does not depend on θ, we obtain that $w^{h\ell} > w^\ell$. This implies two consequences. First, it makes no longer redundant to condition the tax liability also on (reported) income (besides on e). Second, $\epsilon_{w,e}^{h\ell}$ is not necessarily equal to $\epsilon_{w,e}^{\ell}$. In a setting without IM, e^ℓ should be downward- or upward distorted depending on whether $\epsilon_{w,e}^{h\ell}$ is, respectively, larger or smaller than $\epsilon_{w,e}^{\ell}$. With IM the condition for downward distorting e^ℓ becomes $I^{h\ell}\epsilon_{w,e}^{h\ell} > I^\ell \epsilon_{w,e}^{\ell}$, and since one can show that $a^{h\ell} > a^\ell$, it follows that $I^{h\ell} > I^\ell$. To fix ideas, the following Proposition considers the special case when the function $w(e, z)$ is Cobb-Douglas.

Proposition 6 *Assume that the function $w(e, z)$ is Cobb-Douglas. Under an optimal nonlinear tax $T(M, e)$ we have that:*

(i) In a setting without IM, e^ℓ should be left undistorted;

(ii) In a setting with IM, e^ℓ should be downward distorted.

Proof. See the Appendix C. ■

6 Concluding remarks

Within a Mirrleesian optimal tax framework, we have studied the joint design of nonlinear income- and education taxes in a two-type model where an agent's ability-type affects both the (effort) costs and the (pecuniary) benefits of acquiring education. Following several earlier contributions on this topic, we have assumed that educational choices are publicly observable at the individual level; however, we departed from the previous literature by allowing for the possibility that agents conceal part of their earned income for tax purposes.

Using the riskless approach of Usher (1986) to model IM, we have characterized the properties of an informationally constrained Pareto-efficient tax policy, focusing on the so called "normal" case where the direction of redistribution goes from the high- to the low-ability type. We have shown that, when an agent's productivity depends *only indirectly* on his type (i.e., it depends on the individual's type only through the education level chosen by the agent), IM does not affect the qualitative properties of an optimal tax policy. Whether or not earned income is perfectly observable at the individual level, all agents face a zero marginal income tax rate and education is downward distorted for low-ability agents (while it is left undistorted for high-ability agents). A simple intuition for this result comes from the observation that, when the individual hidden characteristic only affects a person's cost of attaining a given education level, a nonlinear tax

[28] See the Appendix.

on education is all that the government needs to control. In particular, even if IM were not an issue, conditioning the tax liability also on earned income would serve no purpose once an optimal nonlinear tax on education is in place.

When an agent's productivity depends, at least partly, *directly* on his type (i.e., when an agent's productivity remains type-dependent even when keeping education fixed), the results are different. IM does not affect the results about the sign of the marginal income tax rate faced by low-ability agents, which should be positive, and high-ability agents, which should be zero. It also leaves unscathed the result that education should be undistorted for high-ability agents. However, compared with a setting where earned income is perfectly observed by the government at the individual level, the case for a downward distortion on the educational choice of low-ability agents is strengthened.

An interesting result that emerges from our study is our finding that IM opens up an avenue for the usefulness of consumption taxation (as long as the productivity of agents depends directly on their types). The reason for it is that with the agents' productivity depending directly on their types, a high-ability mimicker and a low-ability agent differ in the amount of income that they earn, despite the fact that both of them report the same amount to the tax authority. In particular, a high-ability mimicker earns more than a low-ability agent, and therefore can afford a higher consumption. As a consequence, a linear consumption tax imposes a heavier burden on a mimicker than on a low-ability type, and it can be used to soften the binding self-selection constraint faced by the government. Obviously, the same logic implies that consumption taxation would also be welfare-enhancing in a generalized version of our model with several consumption goods. But in such a generalized version we would also obtain a violation of the celebrated Atkinson-Stiglitz (1976) result. Despite the assumption of separability between labor and consumption in the individuals' utility function, differentiated commodity taxation would be a valuable policy instrument (unless the subutility of consumption is homothetic).

The imposition of a linear consumption tax changes the nature of optimal marginal taxes. Its presence implies that any reform of the nonlinear tax on education and reported income that is welfare-neutral for either agent has behavioral effects for that agent (operating via adjustments in IM and thus labor supply). In turn, these effects have an impact on the revenue collected via consumption-taxation. Given that the optimal consumption tax rate is positive, to internalize these fiscal externalities, we show that it becomes desirable to let high-ability agents face a negative marginal tax on reported income and also distort upwards their educational choice. For low-ability agents, the same fiscal-externality considerations imply a moderating effect on the optimal marginal tax on reported income. They also imply a mitigating effect on the tendency, attributable to the possibility of IM, to warrant a downward distortion on the educational choice of low-ability individuals.

The bulk of our analysis has been conducted under the assumption that education is a unidimensional variable that is publicly observable at the individual level. This assumption was relaxed in a final section where we modified the wage function in order to capture the multidimensional nature of human capital investments and allow for the possibility that they are not fully observable by the government. We have highlighted that our qualitative results, and especially the fact that IM strengthens the case for downward distorting the education acquired by low-ability agents, are robust to these modifications of the model assumptions.

Appendix A

Proof of Proposition 1 The first-order conditions of the government's problem are:

$$\frac{v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{w\left(\theta^\ell,e^\ell\right)}=\lambda\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w(\theta^h,e^\ell)}\right)}{w\left(\theta^h,e^\ell\right)}+\mu\pi, \tag{A1}$$

$$\frac{\left(M^\ell+a^\ell\right)\frac{\partial w(\theta^\ell,e^\ell)}{\partial e^\ell}}{\left(w\left(\theta^\ell,e^\ell\right)\right)^2}v'\left(\frac{M^\ell+a^\ell}{w\left(\theta^\ell,e^\ell\right)}\right)-\frac{\partial\varphi\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}=$$

$$\lambda\left[\frac{\left(M^\ell+a^{h\ell}\right)\frac{\partial w(\theta^h,e^\ell)}{\partial e^\ell}}{\left(w\left(\theta^h,e^\ell\right)\right)^2}v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)-\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}\right]+\mu\pi p, \tag{A2}$$

$$u'\left(c^\ell\right)-\lambda u'\left(c^{h\ell}\right)=\mu\pi, \tag{A3}$$

$$\left(\delta+\lambda\right)\frac{v'\left(\frac{M^h+a^h}{w(\theta^h,e^h)}\right)}{w\left(\theta^h,e^h\right)}=\left(1-\pi\right)\mu, \tag{A4}$$

$$\left(\delta+\lambda\right)\left[\frac{\left(M^h+a^h\right)\frac{\partial w(\theta^h,e^h)}{\partial e^h}}{\left(w\left(\theta^h,e^h\right)\right)^2}v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)-\frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h}\right]=\left(1-\pi\right)\mu p, \tag{A5}$$

$$\left(\delta+\lambda\right)u'\left(c^h\right)=\mu\left(1-\pi\right). \tag{A6}$$

From the first-order conditions pertaining to M^h, e^h and B^h (eqs. (A4)-(A6)) we get:

$$1-\frac{v'\left(\frac{M^h+a^h}{w(\theta^h,e^h)}\right)}{w\left(\theta^h,e^h\right)u'\left(c^h\right)}=0,$$

$$\frac{L^h\frac{\partial w(\theta^h,e^h)}{\partial e^h}}{w\left(\theta^h,e^h\right)}\frac{v'\left(L^h\right)}{u'\left(c^h\right)}-\frac{\frac{\partial\varphi(\theta^h,e^h)}{\partial e^h}}{u'\left(c^h\right)}-p=0,$$

which imply, from (9)-(10), that $T_M\left(M^h,e^h\right)=0$ and $T_e\left(M^h,e^h\right)=p$ and therefore, from (11), $a^h=0$.

From the eq. (A1) and eq. (A3), pertaining to the low-skilled, we get

$$1-\frac{v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{w\left(\theta^\ell,e^\ell\right)u'\left(c^\ell\right)}=\frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\frac{v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{w\left(\theta^\ell,e^\ell\right)u'\left(c^\ell\right)}-\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w(\theta^h,e^\ell)}\right)}{w\left(\theta^h,e^\ell\right)u'\left(c^{h\ell}\right)}\right], \tag{A7}$$

from which, by using (9), we obtain (16).

We also get (from eqs. (A2)-(A3)):

$$\left[\frac{\left(M^\ell+a^\ell\right)\frac{\partial w(\theta^\ell,e^\ell)}{\partial e^\ell}}{\left(w\left(\theta^\ell,e^\ell\right)\right)^2}\frac{v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{u'\left(c^\ell\right)}-\frac{\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{u'\left(c^\ell\right)}\right]\left[\lambda u'\left(c^{h\ell}\right)+\mu\pi\right]=$$

$$\lambda\left[\frac{\left(M^\ell+a^{h\ell}\right)\frac{\partial w(\theta^h,e^\ell)}{\partial e^\ell}}{\left(w\left(\theta^h,e^\ell\right)\right)^2}v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)-\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}\right]+\mu\pi p,$$

or equivalently

$$\frac{\left(M^{\ell}+a^{\ell}\right)\frac{\partial w\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{\left(w\left(\theta^{\ell},e^{\ell}\right)\right)^{2}}\frac{v'\left(\frac{M^{\ell}+a^{\ell}}{w\left(\theta^{\ell},e^{\ell}\right)}\right)}{u'\left(c^{\ell}\right)}-\frac{\frac{\partial\varphi\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{u'\left(c^{\ell}\right)}-p=$$

$$\frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\frac{\left(M^{\ell}+a^{h\ell}\right)\frac{\partial w\left(\theta^{h},e^{\ell}\right)}{\partial e^{\ell}}}{w\left(\theta^{h},e^{\ell}\right)}\frac{v'\left(\frac{M^{\ell}+a^{h\ell}}{w\left(\theta^{h},e^{\ell}\right)}\right)}{w\left(\theta^{h},e^{\ell}\right)u'\left(c^{h\ell}\right)}-\frac{\frac{\partial\varphi\left(\theta^{h},e^{\ell}\right)}{\partial e^{\ell}}}{u'\left(c^{h\ell}\right)}\right]=$$

$$-\frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\frac{\left(M^{\ell}+a^{\ell}\right)\frac{\partial w\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{w\left(\theta^{\ell},e^{\ell}\right)}\frac{v'\left(\frac{M^{\ell}+a^{\ell}}{w\left(\theta^{\ell},e^{\ell}\right)}\right)}{w\left(\theta^{\ell},e^{\ell}\right)u'\left(c^{\ell}\right)}-\frac{\frac{\partial\varphi\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{u'\left(c^{\ell}\right)}\right],$$

which can be rewritten as

$$\begin{aligned}&\frac{\left(M^{\ell}+a^{\ell}\right)\frac{\partial w\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{\left(w\left(\theta^{\ell},e^{\ell}\right)\right)^{2}}\frac{v'\left(\frac{M^{\ell}+a^{\ell}}{w\left(\theta^{\ell},e^{\ell}\right)}\right)}{u'\left(c^{\ell}\right)}-\frac{\frac{\partial\varphi\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{u'\left(c^{\ell}\right)}-p=\\&\frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\left(M^{\ell}+a^{h\ell}\right)\epsilon_{w\left(\theta^{h},e^{\ell}\right),e}MRS_{M,B}^{h\ell}-\left(M^{\ell}+a^{\ell}\right)\epsilon_{w\left(\theta^{\ell},e^{\ell}\right),e}MRS_{M,B}^{\ell}\right]\\&+\frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\frac{\frac{\partial\varphi\left(\theta^{\ell},e^{\ell}\right)}{\partial e^{\ell}}}{u'\left(c^{\ell}\right)}-\frac{\frac{\partial\varphi\left(\theta^{h},e^{\ell}\right)}{\partial e^{\ell}}}{u'\left(c^{h\ell}\right)}\right].\end{aligned}\tag{A8}$$

From (A8) one obtains (17) by using (10).

Proof of Lemma 1 *Part (i).* For a given (e, M, B)-bundle, the first-order condition (7) for an optimal choice of a can be rewritten as

$$\left[1-\sigma'(a)\right]u'\left(B+a-\sigma(a)\right)-\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w\left(\theta,e\right)}=0.\tag{A9}$$

It is clear from above that if w is independent of θ, the choice of a will also be independent of θ. This implies that $a^{\ell}=a^{h\ell}$.

Part (ii). If wage depends only on education, from (13)–(14), we have

$$MRS_{MB}^{\ell}=\frac{v'\left(\frac{M^{\ell}+a^{\ell}}{w\left(e^{\ell}\right)}\right)}{w\left(e^{\ell}\right)u'\left(B^{\ell}+a^{\ell}-\sigma\left(a^{\ell}\right)\right)}\text{ and }MRS_{MB}^{h\ell}=\frac{v'\left(\frac{M^{\ell}+a^{h\ell}}{w\left(e^{\ell}\right)}\right)}{w\left(e^{\ell}\right)u'\left(B+a^{h\ell}-\sigma\left(a^{h\ell}\right)\right)}.$$

The equality $a^{\ell}=a^{h\ell}$ then implies that $MRS_{MB}^{h\ell}=MRS_{MB}^{\ell}$.

Part (iii). This follows from the definitions of $\epsilon_{w,e}^{h\ell}=\epsilon_{w,e}^{\ell}$ in (18).

Proof of Lemma 2 *Part (i).* Totally differentiating eq. (A9) gives:

$$\begin{aligned}
&\left[1-\sigma'(a)\right]^2 u''(c)\,da-\sigma''(a)\,u'(c)\,da-\frac{v''(L)}{\left[w(\theta,e)\right]^2}da\\
&= -\frac{w(\theta,e)\,\frac{(M+a)\frac{\partial w(\theta,e)}{\partial\theta}}{[w(\theta,e)]^2}v''(L)+v'(L)\,\frac{\partial w(\theta,e)}{\partial\theta}}{\left[w(\theta,e)\right]^2}d\theta,
\end{aligned}$$

or equivalently,

$$\begin{aligned}
&\left\{\left[1-\sigma'(a)\right]^2 u''(c)-\sigma''(a)\,u'(c)-\frac{v''(L)}{\left[w(\theta,e)\right]^2}\right\}da\\
&= -\frac{\frac{M+a}{w(\theta,e)}v''(L)+v'(L)}{\left[w(\theta,e)\right]^2}\frac{\partial w(\theta,e)}{\partial\theta}d\theta.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\frac{da}{d\theta} &= -\frac{\frac{\frac{M+a}{w(\theta,e)}v''(L)+v'(L)}{[w(\theta,e)]^2}\frac{\partial w(\theta,e)}{\partial\theta}}{\left[1-\sigma'(a)\right]^2 u''(c)-\sigma''(a)\,u'(c)-\frac{v''(L)}{[w(\theta,e)]^2}}\\
&= \frac{\left[Lv''(L)+v'(L)\right]\frac{\frac{\partial w(\theta,e)}{\partial\theta}}{(w(\theta,e))^2}}{\frac{v''(L)}{[w(\theta,e)]^2}+\sigma''(a)\,u'(c)-\left[1-\sigma'(a)\right]^2 u''(c)}. \qquad \text{(A10)}
\end{aligned}$$

Moreover, from the second-order condition of the individual optimization problem, we know that the denominator of the term on the RHS of (A10) is positive; i.e.,

$$\frac{v''(L)}{\left[w(\theta,e)\right]^2}+\sigma''(a)\,u'(c)-\left[1-\sigma'(a)\right]^2 u''(c)>0. \qquad \text{(A11)}$$

It then immediately follows from (A10), (A11), and $v'(\cdot)>0, v''(\cdot)>0$ assumptions that $\partial w(\theta,e)/\partial\theta>0 \implies da/d\theta>0$, so that $a^{h\ell}>a^{\ell}$.

Part (ii). For any given (e,M,B)-bundle, we have

$$MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}=\frac{v'\left(\frac{M+a^{\ell}}{w(\theta^{\ell},e)}\right)}{w\left(\theta^{\ell},e\right)u'\left(B+a^{\ell}-\sigma\left(a^{\ell}\right)\right)}-\frac{v'\left(\frac{M+a^{h\ell}}{w(\theta^{h},e)}\right)}{w\left(\theta^{h},e\right)u'\left(B+a^{h\ell}-\sigma\left(a^{h\ell}\right)\right)}, \qquad \text{(A12)}$$

so that $MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}$ is of opposite sign to $\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'(c)}\right)/\partial\theta$. Differentiating $\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'(c)}$ with respect to θ results in

$$\begin{aligned}
&\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'(c)}\right)}{\partial\theta}=\\
&\frac{w(\theta,e)\,u'(c)\left[\frac{\frac{da}{d\theta}w(\theta,e)-(M+a)\frac{\partial w(\theta,e)}{\partial\theta}}{(w(\theta,e))^2}\right]v''}{\left[w(\theta,e)\,u'(c)\right]^2}-\frac{\left\{\frac{\partial w(\theta,e)}{\partial\theta}u'+w(\theta,e)\left[1-\sigma'(a)\right]\frac{da}{d\theta}u''\right\}v'}{\left[w(\theta,e)\,u'(c)\right]^2}=\\
&\frac{u'v''-w(\theta,e)\left[1-\sigma'(a)\right]u''v'}{\left[w(\theta,e)\,u'(c)\right]^2}\frac{da}{d\theta}-\frac{w(\theta,e)\,u'(c)\,\frac{(M+a)\frac{\partial w(\theta,e)}{\partial\theta}}{(w(\theta,e))^2}v''+\frac{\partial w(\theta,e)}{\partial\theta}u'v'}{\left[w(\theta,e)\,u'(c)\right]^2}. \qquad \text{(A13)}
\end{aligned}$$

Substituting (A10) into (A13) yields

$$\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'}\right)}{\partial\theta} = \frac{u'v''-w(\theta,e)\left[1-\sigma'(a)\right]u''v'}{\left[w(\theta,e)u'\right]^2}\frac{\left[Lv''+v'\right]\frac{\frac{\partial w(\theta,e)}{\partial\theta}}{(w(\theta,e))^2}}{\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)u'-\left[1-\sigma'(a)\right]^2u''}$$
$$-\frac{\frac{M+a}{w(\theta,e)}v''+v'}{\left[w(\theta,e)u'\right]^2}\frac{\partial w(\theta,e)}{\partial\theta}u'.$$

Rearranging the above equation, we have

$$\left[w(\theta,e)u'\right]^2\left[\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)u'-\left[1-\sigma'(a)\right]^2u''\right]\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'}\right)}{\partial\theta}=$$
$$\frac{u'v''-w(\theta,e)\left[1-\sigma'(a)\right]u''v'}{(w(\theta,e))^2}\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}$$
$$-\left[\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)u'-\left[1-\sigma'(a)\right]^2u''\right]\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}u',$$

or equivalently, simplifying terms,

$$\left[w(\theta,e)u'\right]^2\left[\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)u'-\left[1-\sigma'(a)\right]^2u''\right]\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'}\right)}{\partial\theta}=$$
$$\left\{\left[1-\sigma'(a)\right]^2u''-\frac{\left[1-\sigma'(a)\right]u''v'}{w(\theta,e)u'}-\sigma''(a)u'\right\}\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}u'. \tag{A14}$$

Now, from (9) and (11), we have $v'/wu'=1-\sigma'$. This allows us to rewrite (A14) as

$$\left[w(\theta,e)u'\right]^2\left[\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)u'-\left[1-\sigma'(a)\right]^2u''\right]\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'}\right)}{\partial\theta}=$$
$$\left\{\left[1-\sigma'(a)\right]^2u''-\left[1-\sigma'(a)\right]^2u''-\sigma''(a)u'\right\}\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}u',$$

or equivalently, simplifying terms,

$$\left[w(\theta,e)\right]^2\left[\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)u'-\left[1-\sigma'(a)\right]^2u''\right]\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'}\right)}{\partial\theta}=$$
$$-\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}\sigma''(a). \tag{A15}$$

It then follows from (A15), (A11), and $v'(\cdot)>0, v''(\cdot)>0, \sigma''(\cdot)>0$ assumptions, that $\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)u'}\right)/\partial\theta$ and $\partial w(\theta,e)/\partial\theta$ are of opposite signs. Consequently, $MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}$ and $\partial w(\theta,e)/\partial\theta$ are of the same sign, so that $\partial w(\theta,e)/\partial\theta>0 \implies MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}>0$.

Proof of Proposition 2 Dividing (A5) by (A4), and multiplying both sides of the resulting equation by $(1-\pi)\,\mu$ yields

$$\frac{\frac{\left(M^h+a^h\right)\frac{\partial w\left(\theta^h,e^h\right)}{\partial e^h}}{\left(w\left(\theta^h,e^h\right)\right)^2}v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)-\frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h}}{\frac{v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)}}\left(1-\pi\right)\mu=\left(1-\pi\right)\mu p.$$

Simplifying and collecting terms gives

$$\begin{aligned}\frac{\left(M^h+a^h\right)\frac{\partial w\left(\theta^h,e^h\right)}{\partial e^h}}{w\left(\theta^h,e^h\right)}-\frac{w\left(\theta^h,e^h\right)\frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h}}{v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}-p &= \\ L^h\frac{\partial w\left(\theta^h,e^h\right)}{\partial e^h}-\frac{w^h\partial\varphi\left(\theta^h,e^h\right)/\partial e^h}{v'\left(L^h\right)}-p &= 0.\end{aligned}$$

Dividing (A2) by (A1), and multiplying both sides of the resulting equation by the RHS of (A1) gives

$$\frac{\frac{\left(M^\ell+a^\ell\right)\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{\left(w\left(\theta^\ell,e^\ell\right)\right)^2}v'\left(\frac{M^\ell+a^\ell}{w\left(\theta^\ell,e^\ell\right)}\right)-\frac{\partial\varphi\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{\frac{v'\left(\frac{M^\ell+a^\ell}{w\left(\theta^\ell,e^\ell\right)}\right)}{w\left(\theta^\ell,e^\ell\right)}}\left[\lambda\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)}{w\left(\theta^h,e^\ell\right)}+\mu\pi\right]=$$

$$\lambda\left[\frac{\left(M^\ell+a^{h\ell}\right)\frac{\partial w\left(\theta^h,e^\ell\right)}{\partial e^\ell}}{\left(w\left(\theta^h,e^\ell\right)\right)^2}v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)-\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}\right]+\mu\pi p.$$

Rearranging and collecting terms, one can rewrite the above equation as

$$\mu\pi\left[\frac{\left(M^\ell+a^\ell\right)\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{w\left(\theta^\ell,e^\ell\right)}-\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{v'\left(\frac{M^\ell+a^\ell}{w\left(\theta^\ell,e^\ell\right)}\right)}-p\right]=$$

$$\lambda\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)}{w\left(\theta^h,e^\ell\right)}\left[\frac{\left(M^\ell+a^{h\ell}\right)\frac{\partial w\left(\theta^h,e^\ell\right)}{\partial e^\ell}}{w\left(\theta^h,e^\ell\right)}-\frac{\left(M^\ell+a^\ell\right)\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{w\left(\theta^\ell,e^\ell\right)}\right]+$$

$$\lambda\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)}{w\left(\theta^h,e^\ell\right)}\left[\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{v'\left(\frac{M^\ell+a^\ell}{w\left(\theta^\ell,e^\ell\right)}\right)}-\frac{w\left(\theta^h,e^\ell\right)\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}}{v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h,e^\ell\right)}\right)}\right].$$

Dividing both sides by $\mu\pi$ gives:

$$\begin{aligned}&L^\ell\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}-\frac{w^\ell\frac{\partial\varphi\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{v'\left(L^\ell\right)}-p=\\ &\frac{\lambda}{\mu\pi}\frac{v'\left(L^{h\ell}\right)}{w^{h\ell}}\left[I^{h\ell}\epsilon^{h\ell}_{w,e}-I^\ell\epsilon^\ell_{w,e}+\frac{w^\ell\frac{\partial\varphi\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}}{v'\left(L^\ell\right)}-\frac{w^{h\ell}\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}}{v'\left(L^{h\ell}\right)}\right].\end{aligned}$$

Appendix B

Proof of Proposition 3 The first-order conditions of the government's problem are given by

$$\begin{aligned}
& \frac{\left(M^\ell + a^\ell\right)\frac{\partial w\left(\theta^\ell, e^\ell\right)}{\partial e^\ell}}{\left(w\left(\theta^\ell, e^\ell\right)\right)^2} v'\left(\frac{M^\ell + a^\ell}{w\left(\theta^\ell, e^\ell\right)}\right) - \frac{\partial \varphi\left(\theta^\ell, e^\ell\right)}{\partial e^\ell} \\
= {} & \lambda\left[\frac{\left(M^\ell + a^{h\ell}\right)\frac{\partial w\left(\theta^h, e^\ell\right)}{\partial e^\ell}}{\left(w\left(\theta^h, e^\ell\right)\right)^2} v'\left(\frac{M^\ell + a^{h\ell}}{w\left(\theta^h, e^\ell\right)}\right) - \frac{\partial \varphi\left(\theta^h, e^\ell\right)}{\partial e^\ell}\right] \\
& + \mu\pi\left\{p - \frac{t}{1+t}\left[1 - \sigma'\left(a^\ell\right)\right]\frac{da^\ell}{de^\ell}\right\}, \qquad \text{(B1)}
\end{aligned}$$

$$\frac{v'\left(\frac{M^\ell + a^\ell}{w\left(\theta^\ell, e^\ell\right)}\right)}{w\left(\theta^\ell, e^\ell\right)} = \lambda\frac{v'\left(\frac{M^\ell + a^{h\ell}}{w\left(\theta^h, e^\ell\right)}\right)}{w\left(\theta^h, e^\ell\right)} + \mu\pi\left\{1 + \frac{t}{1+t}\left[1 - \sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dM^\ell}\right\}, \qquad \text{(B2)}$$

$$\frac{u'\left(c^\ell\right)}{1+t} = \frac{\lambda u'\left(c^{h\ell}\right)}{1+t} + \mu\pi\left\{1 - \frac{t}{1+t} - \frac{t}{1+t}\left[1 - \sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dB^\ell}\right\}, \qquad \text{(B3)}$$

$$\begin{aligned}
& (\delta + \lambda)\left[\frac{\left(M^h + a^h\right)\frac{\partial w\left(\theta^h, e^h\right)}{\partial e^h}}{\left(w\left(\theta^h, e^h\right)\right)^2} v'\left(\frac{M^h + a^h}{w\left(\theta^h, e^h\right)}\right) - \frac{\partial \varphi\left(\theta^h, e^h\right)}{\partial e^h}\right] \\
= {} & (1 - \pi)\mu\left\{p - \frac{t}{1+t}\left[1 - \sigma'\left(a^h\right)\right]\frac{da^h}{de^h}\right\}, \qquad \text{(B4)}
\end{aligned}$$

$$(\delta + \lambda)\frac{v'\left(\frac{M^h + a^h}{w\left(\theta^h, e^h\right)}\right)}{w\left(\theta^h, e^h\right)} = \mu(1 - \pi)\left\{1 + \frac{t}{1+t}\left[1 - \sigma'\left(a^h\right)\right]\frac{da^h}{dM^h}\right\}, \qquad \text{(B5)}$$

$$\frac{(\delta + \lambda)u'\left(c^h\right)}{1+t} = \mu(1 - \pi)\left\{1 - \frac{t}{1+t} - \frac{t}{1+t}\left[1 - \sigma'\left(a^h\right)\right]\frac{da^h}{dB^h}\right\}, \qquad \text{(B6)}$$

$$\begin{aligned}
& -\frac{c^\ell u'\left(c^\ell\right)}{1+t} - (\delta + \lambda)\frac{c^h u'\left(c^h\right)}{1+t} + \lambda\frac{c^{h\ell} u'\left(c^{h\ell}\right)}{1+t} \\
& + \mu\pi\frac{1}{1+t}\left\{c^\ell + t\left[1 - \sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dt}\right\} + \mu(1 - \pi)\frac{1}{1+t}\left\{c^h + t\left[1 - \sigma'\left(a^h\right)\right]\frac{da^h}{dt}\right\} \\
= {} & 0. \qquad \text{(B7)}
\end{aligned}$$

Equations (B1)–(B7) determine the optimal values of $e^\ell, M^\ell, B^\ell, e^h, M^h, B^h$, and t.

To derive an expression for t, start with multiplying both sides of (B3) by c^ℓ. This gives

$$\frac{u'\left(c^\ell\right)}{1+t}c^\ell = \frac{\lambda u'\left(c^{h\ell}\right)}{1+t}c^\ell + \mu\pi c^\ell\left\{1 - \frac{t}{1+t} - \frac{t}{1+t}\left[1 - \sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dB^\ell}\right\}. \qquad \text{(B8)}$$

Then multiply both sides of (B6) by c^h to get

$$\frac{(\delta + \lambda)u'\left(c^h\right)}{1+t}c^h = \mu(1 - \pi)c^h\left\{1 - \frac{t}{1+t} - \frac{t}{1+t}\left[1 - \sigma'\left(a^h\right)\right]\frac{da^h}{dB^h}\right\}. \qquad \text{(B9)}$$

Substituting for $\frac{u'\left(c^{\ell}\right)}{1+t}c^{\ell}$ and $\frac{(\delta+\lambda)u'\left(c^{h}\right)}{1+t}$ in (B7) the expressions provided by (B8) and (B9) results in

$$\begin{aligned}
& \lambda\frac{u'\left(c^{h\ell}\right)}{1+t}\left(c^{h\ell}-c^{\ell}\right) \\
& +\mu\pi\frac{1}{1+t}\left\{c^{\ell}+t\left[1-\sigma'\left(a^{\ell}\right)\right]\frac{da^{\ell}}{dt}\right\}+\mu\left(1-\pi\right)\frac{1}{1+t}\left\{c^{h}+t\left[1-\sigma'\left(a^{h}\right)\right]\frac{da^{h}}{dt}\right\} \\
= \; & \mu\pi c^{\ell}\left\{1-\frac{t}{1+t}-\frac{t}{1+t}\left[1-\sigma'\left(a^{\ell}\right)\right]\frac{da^{\ell}}{dB^{\ell}}\right\} \\
& +\mu\left(1-\pi\right)c^{h}\left\{1-\frac{t}{1+t}-\frac{t}{1+t}\left[1-\sigma'\left(a^{h}\right)\right]\frac{da^{h}}{dB^{h}}\right\}.
\end{aligned}$$

Simplifying and rearranging terms one obtains

$$\begin{aligned}
& \frac{\lambda}{\mu}u'\left(c^{h\ell}\right)\left(c^{\ell}-c^{h\ell}\right) \\
= \; & \pi t\left[1-\sigma'\left(a^{\ell}\right)\right]\left(\frac{da^{\ell}}{dt}+\frac{da^{\ell}}{dB^{\ell}}c^{\ell}\right)+\left(1-\pi\right)t\left[1-\sigma'\left(a^{h}\right)\right]\left(\frac{da^{h}}{dt}+\frac{da^{h}}{dB^{h}}c^{h}\right),
\end{aligned}$$

from which one gets the result stated in eq. (39).

The properties of $T\left(M,e\right)$ that implements the optimal values of $e^{\ell},M^{\ell},B^{\ell},e^{h},M^{h},B^{h}$ are then found as follows. Faced with $T\left(M,e\right)$, or alternatively $B\left(M,e\right)\equiv M-T\left(M,e\right)$, and a given value for t, an agent solves the following maximization problem

$$\max_{M,e,a}\quad u\left(\frac{M+a-T\left(M,e\right)-\sigma\left(a\right)}{1+t}\right)-v\left(\frac{M+a}{w\left(\theta,e\right)}\right)-\varphi\left(\theta,e\right).$$

The associated first order conditions are:

$$\frac{1-T_M}{1+t}u'-\frac{v'}{w} = 0, \tag{B10}$$

$$\frac{-T_e}{1+t}u'+\frac{\left(M+a\right)\partial w/\partial e}{w^2}v'-\frac{\partial\varphi}{\partial e} = 0, \tag{B11}$$

$$\frac{1-\sigma'}{1+t}u'-\frac{v'}{w} = 0. \tag{B12}$$

From (B10)-(B11), we can derive the following implicit characterizations for the marginal tax rates $T_M\equiv\partial T\left(M,e\right)/\partial M$ and $T_e\equiv\partial T\left(M,e\right)/\partial e$:

$$T_M \equiv \frac{\partial T\left(M,e\right)}{\partial M}=1-\frac{\left(1+t\right)v'}{wu'}, \tag{B13}$$

$$T_e \equiv \frac{\partial T\left(M,e\right)}{\partial e}=\left[\frac{\left(M+a\right)\partial w/\partial e}{w^2}v'-\frac{\partial\varphi}{\partial e}\right]\frac{1+t}{u'}=\left(\frac{L}{w}\frac{\partial w}{\partial e}v'-\frac{\partial\varphi}{\partial e}\right)\frac{1+t}{u'}. \tag{B14}$$

Notice also that, combining (B10) and (B12), one gets that

$$T_M=\sigma'. \tag{B15}$$

Next define the marginal rate of substitution between M and B, denoted by MRS_{MB}, for an agent choosing a given (e, M, B)-bundle. Since for a given (e, M, B)-bundle an agent's indirect utility is given by

$$\max_a u\left(\frac{B+a-\sigma(a)}{1+t}\right) - v\left(\frac{M+a}{w(\theta,e)}\right) - \varphi(\theta,e), \tag{B16}$$

the agent's (conditional) indirect utility is given by

$$V(e, M, B, t; \theta) = u\left(\frac{B+a^*-\sigma(a^*)}{1+t}\right) - v\left(\frac{M+a^*}{w(\theta,e)}\right) - \varphi(\theta,e),$$

where a^* denotes the value for a that solves the problem (B16). By invoking the envelope theorem, we have that $\frac{\partial V}{\partial M} = -\frac{v'\left(\frac{M+a^*}{w(\theta,e)}\right)}{w(\theta,e)}$ and $\frac{\partial V}{\partial B} = \frac{1}{1+t}u'\left(\frac{B+a^*-\sigma(a^*)}{1+t}\right)$. Thus, we can define MRS_{MB} as

$$MRS_{MB} \equiv -\frac{\partial V(e, M, B, t; \theta)}{\partial M} \Big/ \frac{\partial V(e, M, B, t; \theta)}{\partial B} = \frac{(1+t)\, v'\left(\frac{M+a^*}{w(\theta,e)}\right)}{w(\theta,e)\, u'\left(\frac{B+a^*-\sigma(a^*)}{1+t}\right)}. \tag{B17}$$

Based on (B17) we introduce the following notation:

$$MRS_{MB}^{j} = \frac{(1+t)\, v'\left(\frac{M^j+a^j}{w\left(\theta^j, e^j\right)}\right)}{w\left(\theta^j, e^j\right) u'\left(\frac{B^j+a^j-\sigma(a^j)}{1+t}\right)} = \frac{(1+t)\, v'\left(L^j\right)}{w\left(\theta^j, e^j\right) u'\left(c^j\right)} \quad \text{for } j = h, \ell, \tag{B18}$$

$$MRS_{MB}^{h\ell} = \frac{(1+t)\, v'\left(\frac{M^\ell+a^{h\ell}}{w\left(\theta^h, e^\ell\right)}\right)}{w\left(\theta^h, e^\ell\right) u'\left(\frac{B^\ell+a^{h\ell}-\sigma\left(a^{h\ell}\right)}{1+t}\right)} = \frac{(1+t)\, v'\left(L^{h\ell}\right)}{w\left(\theta^h, e^\ell\right) u'\left(c^{h\ell}\right)}. \tag{B19}$$

We now prove that the denominator of the expression for t in eq. (39) is negative so that $sign(t) = sign\left(c^{h\ell} - c^\ell\right)$. To this end, first observe that, from (B13) and (B15), and the fact that $t > -1$, we have $1 - \sigma' > 0$. Next, rewrite (B12) as $(1+t)\, v' = (1-\sigma')\, wu'$; totally differentiating this expression gives

$$\frac{da}{dt} = -\frac{v' + (1-\sigma')\, wu''c/(1+t)}{\frac{(1+t)v''}{w} - \frac{(1-\sigma')^2 wu''}{1+t} + wu'\sigma''}, \tag{B20}$$

$$\frac{da}{dB} = \frac{(1-\sigma')\, wu''/(1+t)}{\frac{(1+t)v''}{w} - \frac{(1-\sigma')^2 wu''}{1+t} + wu'\sigma''} < 0. \tag{B21}$$

It follows from (B20)–(B21) that

$$\frac{da}{dt} + \frac{da}{dB}c = -\frac{v'}{\frac{(1+t)v''}{w} - \frac{(1-\sigma')^2 wu''}{1+t} + wu'\sigma''} < 0,$$

which proves that the denominator of the expression on the RHS of (39) is negative.

To complete the proof, we show below that Lemma 1 and Lemma 2 remain valid in this setting. We do this separately for the two lemmas following the same steps we took there:

Lemma 1 in which $\partial w/\partial\theta = 0$. First, for a given t and a given (e, M, B)-bundle, an agent's optimal choice for a satisfies the first order condition

$$(1+t)\, v'\left(\frac{M+a}{w(\theta,e)}\right) = w(\theta,e)\left[1-\sigma'(a)\right] u'\left(\frac{B+a-\sigma(a)}{1+t}\right). \tag{B22}$$

It is clear from above that if w is independent of θ, the choice of a will also be independent of θ. This implies that $a^{\ell} = a^{h\ell}$.

Second, it follows from (B18)–(B19) that, with $a^{h\ell} = a^{\ell}$ (which implies $L^{h\ell} = L^{\ell}, c^{h\ell} = c^{\ell}$ and w independent of θ, $MRS^{\ell}_{MB} = MRS^{h\ell}_{MB}$.

Third, the result continues to hold from definitions of $\epsilon^{h\ell}_{w,e} = \epsilon^{\ell}_{w,e}$ in (18).

Lemma 2 in which $\partial w(\theta,e)/\partial\theta > 0$. To prove the validity of the first result of this Lemma, totally differentiate eq. (B22) to get

$$\begin{aligned}
&(1+t)\left[v''\left(\frac{M+a}{w(\theta,e)}\right)\frac{da}{w(\theta,e)} - v''\left(\frac{M+a}{w(\theta,e)}\right)\frac{M+a}{(w(\theta,e))^2}\frac{\partial w(\theta,e)}{\partial\theta}d\theta\right] = \\
&w(\theta,e)\left\{-\sigma''(a)\,u'\left(\frac{B+a-\sigma(a)}{1+t}\right)da + \left[1-\sigma'(a)\right]^2 u''\left(\frac{B+a-\sigma(a)}{1+t}\right)\frac{1}{1+t}da\right\} \\
&+\left[1-\sigma'(a)\right] u'\left(\frac{B+a-\sigma(a)}{1+t}\right)\frac{\partial w(\theta,e)}{\partial\theta}d\theta.
\end{aligned}$$

From eq. (B22), substitute $\frac{1+t}{w(\theta,e)}v'\left(\frac{M+a}{w(\theta,e)}\right)$ for $\left[1-\sigma'(a)\right]u'\left(\frac{B+a-\sigma(a)}{1+t}\right)$ in above, then rearrange and simplify to get:

$$\begin{aligned}
&\left\{\frac{1}{(w(\theta,e))^2}v''\left(\frac{M+a}{w(\theta,e)}\right) + \frac{\sigma''(a)}{1+t}u'\left(\frac{B+a-\sigma(a)}{1+t}\right) - \frac{\left[1-\sigma'(a)\right]^2}{(1+t)^2}u''\left(\frac{B+a-\sigma(a)}{1+t}\right)\right\}da \\
&= \left\{v'\left(\frac{M+a}{w(\theta,e)}\right) + \frac{M+a}{w(\theta,e)}v''\left(\frac{M+a}{w(\theta,e)}\right)\right\}\frac{1}{(w(\theta,e))^2}\frac{\partial w(\theta,e)}{\partial\theta}d\theta.
\end{aligned}$$

We thus have,

$$\begin{aligned}
\frac{da}{d\theta} &= \frac{v'\left(\frac{M+a}{w(\theta,e)}\right) + \frac{M+a}{w(\theta,e)}v''\left(\frac{M+a}{w(\theta,e)}\right)}{(w(\theta,e))^2\left\{\frac{1}{(w(\theta,e))^2}v''\left(\frac{M+a}{w(\theta,e)}\right) + \frac{\sigma''(a)}{1+t}u'\left(\frac{B+a-\sigma(a)}{1+t}\right) - \frac{[1-\sigma'(a)]^2}{(1+t)^2}u''\left(\frac{B+a-\sigma(a)}{1+t}\right)\right\}}\frac{\partial w(\theta,e)}{\partial\theta} \\
&= \frac{v'(L) + Lv''(L)}{(w(\theta,e))^2\left\{\frac{1}{(w(\theta,e))^2}v''(L) + \frac{\sigma''(a)}{1+t}u'(c) - \frac{[1-\sigma'(a)]^2}{(1+t)^2}u''(c)\right\}}\frac{\partial w(\theta,e)}{\partial\theta}.
\end{aligned} \tag{B23}$$

From the second-order condition of the individual optimization problem, we know that

$$\frac{1}{(w(\theta,e))^2}v''(L) + \frac{\sigma''(a)}{1+t}u'(c) - \frac{\left[1-\sigma'(a)\right]^2}{(1+t)^2}u''(c) > 0. \tag{B24}$$

From (B23), (B24), and $v'(\cdot) > 0, v''(\cdot) > 0$ assumptions, it follows that $\partial w(\theta,e)/\partial\theta > 0 \implies da/d\theta > 0$, so that $a^{h\ell} > a^{\ell}$.

Second, define τ as $\tau \equiv 1/(1+t)$. For any given (e, M, B)-bundle we have, from (B18)–(B19), that

$$MRS^{\ell}_{MB} - MRS^{h\ell}_{MB} = \frac{1}{\tau}\left[\frac{v'\left(\frac{M+a^{\ell}}{w(\theta^{\ell},e)}\right)}{w(\theta^{\ell},e)\,u'\left((B+a^{\ell}-\sigma(a^{\ell}))\tau\right)} - \frac{v'\left(\frac{M+a^{h\ell}}{w(\theta^{h},e)}\right)}{w(\theta^{h},e)\,u'\left((B+a^{h\ell}-\sigma(a^{h\ell}))\tau\right)}\right],$$

so that $MRS^{\ell}_{MB} - MRS^{h\ell}_{MB}$ is of opposite sign to $\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'(c)}\right)/\partial\theta$. Differentiating $\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'(c)}$ with respect to θ results in

$$\begin{aligned}\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'(c)}\right)}{\partial\theta} &= \frac{\tau u'v'' - w(\theta,e)\left[1-\sigma'(a)\right]\tau^2 u''v'}{\left[w(\theta,e)\tau u'(c)\right]^2}\frac{da}{d\theta} \\ &\quad - \frac{w(\theta,e)\tau u'(c)\frac{(M+a)\frac{\partial w(\theta,e)}{\partial\theta}}{(w(\theta,e))^2}v'' + \frac{\partial w(\theta,e)}{\partial\theta}\tau u'v'}{\left[w(\theta,e)\tau u'(c)\right]^2}. \end{aligned} \tag{B25}$$

Replace $1/(1+t)$ by τ in the expression for $da/d\theta$, given by (B23), and substitute from it into (B25) to get

$$\begin{aligned}&\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'(c)}\right)}{\partial\theta} \\ =\ &\frac{\tau u'v'' - w(\theta,e)\left[1-\sigma'(a)\right]\tau^2 u''v'}{\left[w(\theta,e)\tau u'(c)\right]^2}\frac{\left[Lv''(L)+v'(L)\right]\frac{\frac{\partial w(\theta,e)}{\partial\theta}}{(w(\theta,e))^2}}{\frac{v''(L)}{[w(\theta,e)]^2}+\sigma''(a)\tau u'(c) - \left[1-\sigma'(a)\right]^2\tau^2 u''(c)} \\ &-\frac{\frac{M+a}{w(\theta,e)}v''+v'}{\left[w(\theta,e)\tau u'(c)\right]^2}\frac{\partial w(\theta,e)}{\partial\theta}\tau u'.\end{aligned}$$

From the above equation above we can also write

$$\begin{aligned}&\left[w(\theta,e)\tau u'\right]^2\left\{\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)\tau u' - \left[1-\sigma'(a)\right]^2\tau^2 u''\right\}\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'}\right)}{\partial\theta} \\ =\ &\left\{\frac{\tau u'v'' - w(\theta,e)\left[1-\sigma'(a)\right]\tau^2 u''v'}{(w(\theta,e))^2}\right\}\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta} \\ &-\left\{\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)\tau u' - \left[1-\sigma'(a)\right]^2\tau^2 u''\right\}\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}\tau u',\end{aligned}$$

or equivalently, simplifying terms,

$$\begin{aligned}&\left[w(\theta,e)\tau u'\right]^2\left\{\frac{v''}{\left[w(\theta,e)\right]^2}+\sigma''(a)\tau u' - \left[1-\sigma'(a)\right]^2\tau^2 u''\right\}\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'}\right)}{\partial\theta} \\ =\ &\left\{\left[1-\sigma'(a)\right]^2\tau^2 u'' - \frac{\left[1-\sigma'(a)\right]\tau u''v'}{w(\theta,e)u'} - \sigma''(a)\tau u'\right\}\left[Lv''+v'\right]\frac{\partial w(\theta,e)}{\partial\theta}\tau u'.\end{aligned} \tag{B26}$$

Now notice that, exploiting (B15) we have that $v'/(w\tau u') = 1-\sigma'$, which implies that we can rewrite (B26) as

$$
\begin{aligned}
& \left[w\left(\theta,e\right)\tau u'\right]^2\left\{\frac{v''}{\left[w\left(\theta,e\right)\right]^2}+\sigma''\left(a\right)\tau u'-\left[1-\sigma'\left(a\right)\right]^2\tau^2u''\right\}\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'}\right)}{\partial\theta} \\
= & \left\{\left[1-\sigma'\left(a\right)\right]^2\tau^2u''-\left[1-\sigma'\left(a\right)\right]^2\tau^2u''-\sigma''\left(a\right)\tau u'\right\}\left[Lv''+v'\right]\frac{\partial w\left(\theta,e\right)}{\partial\theta}\tau u',
\end{aligned}
$$

or equivalently, simplifying terms:

$$
\begin{aligned}
& \left[w\left(\theta,e\right)\right]^2\left\{\frac{v''}{\left[w\left(\theta,e\right)\right]^2}+\sigma''\left(a\right)\tau u'-\left[1-\sigma'\left(a\right)\right]^2\tau^2u''\right\}\frac{\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'}\right)}{\partial\theta} \\
= & -\left[Lv''+v'\right]\frac{\partial w\left(\theta,e\right)}{\partial\theta}\sigma''\left(a\right). \qquad \text{(B27)}
\end{aligned}
$$

Next, we showed in above that $MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}$ is of opposite sign to $\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'(c)}\right)/\partial\theta$. It also follows from (B27), (B24), and $v'\left(\cdot\right)>0, v''\left(\cdot\right)>0, \sigma''\left(\cdot\right)>0$ assumptions, that $\partial\left(\frac{v'\left(\frac{M+a}{w(\theta,e)}\right)}{w(\theta,e)\tau u'(c)}\right)/\partial\theta$ and $\partial w\left(\theta,e\right)/\partial\theta$ are of opposite signs. Consequently, $MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}$ and $\partial w\left(\theta,e\right)/\partial\theta$ are of the same sign, so that $\partial w\left(\theta,e\right)/\partial\theta>0 \implies MRS^{\ell}_{MB}-MRS^{h\ell}_{MB}>0$.

Proof of Proposition 4 (i) Derivation of the expression for $T_M\left(M^h,e^h\right)$ in (41). Dividing (B5) by (B6) and multiplying both sides of the resulting equation by the RHS of (B6) gives

$$
\frac{\frac{v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)}}{\frac{u'\left(c^h\right)}{1+t}}\left\{1-\frac{t}{1+t}-\frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dB^h}\right\}=\left\{1+\frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dM^h}\right\}.
$$

Rearranging and collecting terms one can rewrite the equation above as

$$
\begin{aligned}
& 1-\frac{\left(1+t\right)v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)u'\left(c^h\right)} \\
= & -\frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dM^h}-\frac{t}{1+t}\frac{\frac{v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)}}{\frac{u'\left(c^h\right)}{1+t}}\left\{1+\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dB^h}\right\}.
\end{aligned}
$$

Exploiting (B13) the equation above can be restated as

$$
\begin{aligned}
T_M\left(M^h,e^h\right) = & -\frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\left[\frac{da^h}{dM^h}+\frac{\left(1+t\right)v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)u'\left(c^h\right)}\frac{da^h}{dB^h}\right] \\
& -\frac{t}{1+t}\frac{\left(1+t\right)v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)u'\left(c^h\right)},
\end{aligned}
$$

from which one obtains (41) by applying (B18).

(ii) Derivation of the expression for $T_M\left(M^\ell, e^\ell\right)$ in (42). Dividing (B2) by (B3) and multiplying both sides of the resulting equation by the RHS of (B3) gives

$$\begin{aligned}
&\frac{(1+t)\,v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{w\left(\theta^\ell,e^\ell\right)u'\left(c^\ell\right)}\left\{\frac{\lambda u'\left(c^{h\ell}\right)}{1+t}+\mu\pi\left[1-\frac{t}{1+t}-\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{da^\ell}{dB^\ell}\right]\right\}\\
=\;&\lambda\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w(\theta^h,e^\ell)}\right)}{w\left(\theta^h,e^\ell\right)}+\mu\pi\left\{1+\frac{t}{1+t}\left[1-\sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dM^\ell}\right\}.
\end{aligned}$$

Rearranging and collecting terms one can rewrite the equation above as

$$\begin{aligned}
&1-\frac{(1+t)\,v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{w\left(\theta^\ell,e^\ell\right)u'\left(c^\ell\right)}\\
=\;&\frac{\lambda u'\left(c^{h\ell}\right)}{\mu\pi}\left[\frac{v'\left(\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}\right)}{w\left(\theta^\ell,e^\ell\right)u'\left(c^\ell\right)}-\frac{v'\left(\frac{M^\ell+a^{h\ell}}{w(\theta^h,e^\ell)}\right)}{w\left(\theta^h,e^\ell\right)u'\left(c^{h\ell}\right)}\right]\\
&-\frac{t}{1+t}\left[1-\sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dM^\ell}-\left[\frac{t}{1+t}+\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{da^\ell}{dB^\ell}\right]MRS_{MB}^\ell.
\end{aligned}$$

Exploiting (B13) and (B18)-(B19) the equation above can be restated as

$$\begin{aligned}
T_M\left(M^\ell,e^\ell\right)\;=\;&\frac{\lambda u'\left(c^{h\ell}\right)}{(1+t)\,\mu\pi}\left[MRS_{MB}^\ell-MRS_{MB}^{h\ell}\right]\\
&-\frac{t}{1+t}\left[1-\sigma'\left(a^\ell\right)\right]\frac{da^\ell}{dM^\ell}-\left[\frac{t}{1+t}+\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{da^\ell}{dB^\ell}\right]MRS_{MB}^\ell,
\end{aligned}$$

from which the result stated in eq. (42) is obtained.

(iii) $MRS_{MB}+[1-\sigma'][(da/dM)+MRS_{MB}(da/dB)]$ **is positive.** Totally differentiate (B22) to get

$$\frac{da}{dM}=-\frac{(1+t)\,v''/w}{\frac{(1+t)v''}{w}-\frac{(1-\sigma')^2wu''}{1+t}+wu'\sigma''}. \tag{B28}$$

Combining (B21) and (B28) results in

$$\frac{da}{dM}+MRS_{MB}\frac{da}{dB}=\frac{\frac{u''}{u'}v'-\frac{u'}{v'}v''}{\frac{(1+t)v''}{w}-\frac{(1-\sigma')^2wu''}{1+t}+wu'\sigma''}\left(1-\sigma'\right), \tag{B29}$$

$$MRS_{MB}+\left(1-\sigma'\right)\left(\frac{da}{dM}+MRS_{MB}\frac{da}{dB}\right)=\frac{\left[wu'\sigma''-\sigma'\frac{v'}{u'}u''\right](1-\sigma')}{\frac{(1+t)v''}{w}-\frac{(1-\sigma')^2wu''}{1+t}+wu'\sigma''}. \tag{B30}$$

Simplify notation by defining

$$\Omega\;\equiv\;\frac{(1+t)\,v''}{w}+\left[u'\sigma''-\frac{(1-\sigma')\,u''}{1+t}\right]w, \tag{B31}$$

$$\Phi\;\equiv\;MRS_{MB}+\left(1-\sigma'\right)\left(\frac{da}{dM}+MRS_{MB}\frac{da}{dB}\right)=\frac{wu'\sigma''-\sigma'\frac{v'}{u'}u''}{\Omega}\left(1-\sigma'\right), \tag{B32}$$

where the second-order conditions of the individual optimization problem implies that $\Omega > 0$.

To determine the sign of Φ, observe that from (B13) and (B15), and the fact that $t > -1$, we have $1 - \sigma' > 0$. Consequently,

$$sign\left(\Phi\right) = sign\left[\underbrace{wu'\left(c\right)\sigma''\left(a\right)}_{>0} - \underbrace{\sigma'\left(a\right)v'\left(L\right)u''\left(c\right)/u'\left(c\right)}_{?}\right]. \tag{B33}$$

Now, unless preferences are quasi-linear in consumption ($u'' = 0$), Φ appears to be ambiguous in sign. The reason is that $\sigma'\left(a\right)$ could in principle be either positive (when the agent faces a positive marginal tax on reported income, and therefore under-reports his income) or negative (when the agent faces a negative marginal tax on reported income, and therefore over-reports his income). Nevertheless, we can safely establish that $\Phi > 0$.

It is clear from the expression for Φ in (B33), and with $u''\left(c\right) < 0$, that the only possibility under which $\Phi < 0$ is for $\sigma'\left(a\right) < 0$ to make the second term on the RHS of (B33) positive, but also larger in size than the first term. Suppose this is the case. Then, given that $t > 0$ at an optimum, the RHS of (41) which is of opposite sign to Φ, will be positive so that $T_M\left(M^h, e^h\right) > 0$. The same argument tells us that if $\Phi < 0$, the second expression on the RHS of (42) is positive. Moreover, from Lemma 2, we know that the first term on the RHS of (42) is also positive. Consequently, $T_M\left(M^\ell, e^\ell\right) > 0$. This cannot happen though. With $\sigma'\left(a\right) < 0$, condition (B15) implies that $T_M\left(M^j, e^j\right) = \sigma'\left(a^j\right) < 0, j = h, \ell$. A contradiction.

Finally, notice that even though $\Phi > 0$, we have that $\Phi < MRS_{MB}$. This is because from (B29) we have that

$$\left(1-\sigma'\right)\left(\frac{da}{dM} + MRS_{MB}\frac{da}{dB}\right) = \frac{\frac{u''}{u'}v' - \frac{u'}{v'}v''}{\frac{(1+t)v''}{w} - \frac{(1-\sigma')^2 wu''}{1+t} + wu'\sigma''}\left(1-\sigma'\right)^2 < 0.$$

Proof of Proposition 5 (i) Deriving the expression for (47) Dividing (B4) by (B5) and multiplying both sides of the resulting equation by the RHS of (B5) gives:

$$\frac{\left[\frac{\left(M^h+a^h\right)\frac{\partial w\left(\theta^h,e^h\right)}{\partial e^h}}{\left(w\left(\theta^h,e^h\right)\right)^2}v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right) - \frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h}\right]}{\frac{v'\left(\frac{M^h+a^h}{w\left(\theta^h,e^h\right)}\right)}{w\left(\theta^h,e^h\right)}}\left\{1 + \frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dM^h}\right\}$$
$$= p - \frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{de^h}.$$

Simplifying terms allows rewriting the above equation as

$$\left[L^h\frac{\partial w\left(\theta^h,e^h\right)}{\partial e^h} - \frac{w\left(\theta^h,e^h\right)\frac{\partial\varphi\left(\theta^h,e^h\right)}{\partial e^h}}{v'\left(L^h\right)}\right]\left\{1 + \frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{dM^h}\right\}$$
$$= p - \frac{t}{1+t}\left[1-\sigma'\left(a^h\right)\right]\frac{da^h}{de^h}.$$

Rearranging terms gives eq. (47).

(ii) Deriving the expression for (48) Divide (B1) by (B2) and multiply both sides of the resulting equation by the RHS of (B2). One gets, taking into account that $\frac{M^\ell+a^\ell}{w(\theta^\ell,e^\ell)}=L^\ell$ and $\frac{M^\ell+a^{h\ell}}{w(\theta^h,e^\ell)}=L^{h\ell}$,

$$\begin{aligned}
&\frac{\frac{L^\ell\frac{\partial w(\theta^\ell,e^\ell)}{\partial e^\ell}}{w(\theta^\ell,e^\ell)}v'\left(L^\ell\right)-\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{\frac{v'(L^\ell)}{w(\theta^\ell,e^\ell)}}\left[\lambda\frac{v'\left(L^{h\ell}\right)}{w\left(\theta^h,e^\ell\right)}+\mu\pi\left(1+\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{\partial a^\ell}{\partial M^\ell}\right)\right]\\
=\;&\lambda\left[\frac{L^{h\ell}\frac{\partial w(\theta^h,e^\ell)}{\partial e^\ell}}{w\left(\theta^h,e^\ell\right)}v'\left(L^{h\ell}\right)-\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}\right]+\mu\pi\left(p-\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{\partial a^\ell}{\partial e^\ell}\right).
\end{aligned}$$

Simplifying and rearranging terms allows rewriting the equation above as

$$\begin{aligned}
&\left[L^\ell\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{v'\left(L^\ell\right)}\right]\left\{\lambda\frac{v'\left(L^{h\ell}\right)}{w\left(\theta^h,e^\ell\right)}+\mu\pi\left[1+\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{\partial a^\ell}{\partial M^\ell}\right]\right\}\\
=\;&\lambda\left[\frac{L^{h\ell}}{w\left(\theta^h,e^\ell\right)}\frac{\partial w\left(\theta^h,e^\ell\right)}{\partial e^\ell}v'\left(L^{h\ell}\right)-\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}\right]+\mu\pi\left[p-\frac{t}{1+t}\left(1-\sigma'\left(a^\ell\right)\right)\frac{\partial a^\ell}{\partial e^\ell}\right],
\end{aligned}$$

or equivalently

$$\begin{aligned}
&L^\ell\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{v'\left(L^\ell\right)}-p\\
=\;&\frac{\lambda}{\mu\pi}\frac{v'\left(L^{h\ell}\right)}{w\left(\theta^h,e^\ell\right)}\left[L^{h\ell}\frac{\partial w\left(\theta^h,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^h,e^\ell\right)}{v'\left(L^{h\ell}\right)}\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell}\right]\\
&-\frac{\lambda}{\mu\pi}\frac{v'\left(L^{h\ell}\right)}{w\left(\theta^h,e^\ell\right)}\left[L^\ell\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{v'\left(L^\ell\right)}\right]\\
&-\frac{t}{1+t}\left[1-\sigma'\left(a^\ell\right)\right]\left\{\frac{\partial a^\ell}{\partial e^\ell}+\frac{\partial a^\ell}{\partial M^\ell}\left[L^\ell\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{v'\left(L^\ell\right)}\right]\right\}.
\end{aligned}$$

Eq. (48) follows from the above equation once one takes into account that,

$$\begin{aligned}
MRS_{eM}^{h\ell}&\equiv L^{h\ell}\frac{\partial w\left(\theta^h,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^h,e^\ell\right)}{v'\left(L^{h\ell}\right)}\frac{\partial\varphi\left(\theta^h,e^\ell\right)}{\partial e^\ell},\\
MRS_{eM}^{\ell}&\equiv L^{\ell}\frac{\partial w\left(\theta^\ell,e^\ell\right)}{\partial e^\ell}-\frac{w\left(\theta^\ell,e^\ell\right)\frac{\partial\varphi(\theta^\ell,e^\ell)}{\partial e^\ell}}{v'\left(L^{\ell}\right)}.
\end{aligned}$$

(iii) Determining the sign of the expression in (49). Totally differentiate (B22) to get,

$$\frac{da}{de}=\frac{\left(1-\sigma'\right)u'+\left(1+t\right)v''\frac{L}{w}}{\frac{(1+t)v''}{w}-\frac{(1-\sigma')wu''}{1+t}+wu'\sigma''}\frac{\partial w}{\partial e}>0. \tag{B34}$$

It follows from (B28) and (B34), that

$$\frac{da}{de}+\frac{da}{dM}MRS_{eM}^{\ell}=\frac{\left(1-\sigma'\right)u'\frac{\partial w}{\partial e}+\left(1+t\right)\frac{\partial\varphi(\theta,e)}{\partial e}v''/v'}{\frac{(1+t)v''}{w}-\frac{(1-\sigma')^2wu''}{1+t}+wu'\sigma''}>0.$$

Appendix C

Alternative specification of the wage function For a given (e, M, B)-bundle, an agent of ability θ solves the following problem:

$$\max_{a,z} \quad u\left(B + a - \sigma\left(a\right)\right) - v\left(\frac{M + a}{w\left(\theta, e, z\right)}\right) - \varphi\left(\theta, z\right)$$

The associated first order conditions are given by:

$$\left(1 - \sigma'\right) u' - v'/w = 0, \tag{C1}$$

$$\frac{L}{w}\frac{\partial w}{\partial z}v' - \frac{\partial \varphi}{\partial z} = 0. \tag{C2}$$

Totally differentiating (C1)-(C2) gives:

$$\left[\left(1 - \sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2}\right] da + \left(Lv'' + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}dz = -\left(Lv'' + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial \theta}d\theta \tag{C3}$$

$$\begin{aligned} &\left(Lv'' + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}da + \left[\frac{L}{w}\frac{\partial^2 w}{\partial z \partial z}v' - \left(Lv'' + 2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L - \frac{\partial^2 \varphi}{\partial z \partial z}\right] dz \\ = &\left[-\frac{L}{w}\frac{\partial^2 w}{\partial z \partial \theta}v' + \left(Lv'' + 2v'\right)\frac{L}{w^2}\frac{\partial w}{\partial z}\frac{\partial w}{\partial \theta} + \frac{\partial^2 \varphi}{\partial z \partial \theta}\right] d\theta. \end{aligned} \tag{C4}$$

Rewriting (C3)-(C4) in matrix form we have:

$$\begin{aligned} &\begin{bmatrix} \left(1 - \sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2} & \left(Lv'' + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z} \\ \left(Lv'' + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z} & \frac{L}{w}v'\frac{\partial^2 w}{\partial z \partial z} - \left(Lv'' + 2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L - \frac{\partial^2 \varphi}{\partial z \partial z} \end{bmatrix} \begin{bmatrix} da/d\theta \\ dz/d\theta \end{bmatrix} \\ = &\begin{bmatrix} -\left(Lv'' + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial \theta} \\ -\frac{L}{w}\frac{\partial^2 w}{\partial z \partial \theta}v' + \left(Lv'' + 2v'\right)\frac{L}{w^2}\frac{\partial w}{\partial z}\frac{\partial w}{\partial \theta} + \frac{\partial^2 \varphi}{\partial z \partial \theta} \end{bmatrix}. \end{aligned} \tag{C5}$$

Defining by Υ the determinant of the 2x2 matrix on the LHS of (C5), i.e.

$$\begin{aligned} \Upsilon \equiv &\left[\left(1 - \sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2}\right]\left[\frac{L}{w}v'\frac{\partial^2 w}{\partial z \partial z} - \left(Lv'' + 2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L - \frac{\partial^2 \varphi}{\partial z \partial z}\right] \\ &- \left[\left(v'' L + v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}\right]^2, \end{aligned}$$

we have that $\Upsilon > 0$ from the second order conditions of the agent's problem.

Thus, assuming that the second order conditions are satisfied, we have that the sign of $dz/d\theta$ is given by the sign of

$$\begin{aligned} &\underbrace{\left[\left(1 - \sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2}\right]\left(\frac{\partial^2 \varphi}{\partial z \partial \theta} - \frac{L}{w}\frac{\partial^2 w}{\partial z \partial \theta}v'\right)}_{>0} \\ &+ \underbrace{\left[\left(1 - \sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2}\right]\left(Lv'' + 2v'\right)\frac{L}{w^2}\frac{\partial w}{\partial z}\frac{\partial w}{\partial \theta}}_{<0} \\ &+ \underbrace{\left[\left(v'' L + v'\right)\frac{1}{w^2}\right]^2 \frac{\partial w}{\partial z}\frac{\partial w}{\partial \theta}}_{>0}. \end{aligned} \tag{C6}$$

From the second order conditions of the agent's problem we know that

$$\left(1-\sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2} < \frac{\left[\left(v''L+v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}\right]^2}{\frac{L}{w}v'\frac{\partial^2 w}{\partial z\partial z} - \left(Lv''+2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L - \frac{\partial^2 \varphi}{\partial z\partial z}},$$

which in turn implies that, given that $\frac{L}{w}v'\frac{\partial^2 w}{\partial z\partial z} - \frac{\partial^2 \varphi}{\partial z\partial z} < 0$,

$$\left(1-\sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2} < -\frac{\left[\left(v''L+v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}\right]^2}{\left(Lv''+2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L}. \tag{C7}$$

One can then show that $dz/d\theta > 0$. In fact, replacing the second line of (C6) with the RHS of (C7) gives

$$\begin{aligned}&\left[\left(1-\sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2}\right]\left(\frac{\partial^2 \varphi}{\partial z\partial\theta} - \frac{L}{w}\frac{\partial^2 w}{\partial z\partial\theta}v'\right)\\ &-\frac{\left[\left(v''L+v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}\right]^2}{\left(Lv''+2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L}\left(Lv''+2v'\right)\frac{L}{w^2}\frac{\partial w}{\partial z}\frac{\partial w}{\partial\theta}\\ &+\left[\left(v''L+v'\right)\frac{1}{w^2}\right]^2\frac{\partial w}{\partial z}\frac{\partial w}{\partial\theta},\end{aligned}$$

which, simplifying terms, can be rewritten as

$$\left[\left(1-\sigma'\right)^2 u'' - \sigma'' u' - \frac{v''}{w^2}\right]\left(\frac{\partial^2 \varphi}{\partial z\partial\theta} - \frac{L}{w}\frac{\partial^2 w}{\partial z\partial\theta}v'\right) > 0.$$

The fact that $dz/d\theta > 0$ means that our model of Sections 2-4 is equivalent to a model where e and z are perfect complements in the function $w(\theta, e, z)$. The reason is that, for a given (e, M, B)-bundle, both low- and high-ability agents would optimally set $z = e$ under perfect complementarity, which in turn implies that $z^{h\ell} = z^{\ell} = e^{\ell}$.

Consider now the sign of $da/d\theta$. Assuming that the second order conditions of the individual's problem are satisfied, we have that the sign of $da/d\theta$ is given by the sign of

$$\begin{aligned}&-\left(Lv''+v'\right)\frac{1}{w^2}\frac{\partial w}{\partial\theta}\left[\frac{L}{w}v'\frac{\partial^2 w}{\partial z\partial z} - \left(Lv''+2v'\right)\left(\frac{1}{w}\frac{\partial w}{\partial z}\right)^2 L - \frac{\partial^2 \varphi}{\partial z\partial z}\right]\\ &-\left(Lv''+v'\right)\frac{1}{w^2}\frac{\partial w}{\partial z}\left[\frac{\partial^2 \varphi}{\partial z\partial\theta} - \frac{L}{w}\frac{\partial^2 w}{\partial z\partial\theta}v' + \left(Lv''+2v'\right)\frac{L}{w^2}\frac{\partial w}{\partial z}\frac{\partial w}{\partial\theta}\right].\end{aligned} \tag{C8}$$

Simplifying terms, expression (C8) can be rewritten as

$$-\left(Lv''+v'\right)\frac{1}{w^2}\left[\left(\frac{L}{w}v'\frac{\partial^2 w}{\partial z\partial z} - \frac{\partial^2 \varphi}{\partial z\partial z}\right)\frac{\partial w}{\partial\theta} + \left(\frac{\partial^2 \varphi}{\partial z\partial\theta} - \frac{L}{w}\frac{\partial^2 w}{\partial z\partial\theta}v'\right)\frac{\partial w}{\partial z}\right], \tag{C9}$$

which is strictly greater than zero under the assumptions that $\frac{\partial w}{\partial\theta} > 0$, $\frac{\partial w}{\partial z} > 0$, $\frac{\partial^2 w}{\partial z\partial z} < 0$, $\frac{\partial^2 \varphi}{\partial z\partial z} > 0$, $\frac{\partial^2 \varphi}{\partial z\partial\theta} < 0$, $\frac{\partial^2 w}{\partial z\partial\theta} > 0$.

Having established that $da/d\theta > 0$, we can conclude that $I^{h\ell} > I^{\ell}$.

Proof of Proposition 6 The government's problem is

$$\max_{e^\ell, M^\ell, B^\ell, e^h, M^h, B^h} \quad u\left(B^\ell + a^\ell - \sigma\left(a^\ell\right)\right) - v\left(\frac{M^\ell + a^\ell}{w\left(e^\ell, z^\ell\right)}\right) - \varphi\left(\theta^\ell, z^\ell\right)$$

subject to the constraints

$$u\left(B^h + a^h - \sigma\left(a^h\right)\right) - v\left(\frac{M^h + a^h}{w\left(e^h, z^h\right)}\right) - \varphi\left(\theta^h, z^h\right) \geq \overline{V},$$

$$\begin{aligned} & u\left(B^h + a^h - \sigma\left(a^h\right)\right) - v\left(\frac{M^h + a^h}{w\left(e^h, z^h\right)}\right) - \varphi\left(\theta^h, z^h\right) \\ \geq\ & u\left(B^\ell + a^{h\ell} - \sigma\left(a^{h\ell}\right)\right) - v\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right) - \varphi\left(\theta^h, z^{h\ell}\right), \end{aligned}$$

$$\pi\left(M^\ell - B^\ell - pe^\ell\right) + (1-\pi)\left(M^h - B^h - pe^h\right) \geq \overline{R}.$$

Denote respectively by δ, λ and μ the Lagrange multipliers attached to the first-, second- and third constraint. For our purposes only the only relevant first order conditions are those with respect to e^ℓ and M^ℓ. These are respectively given by:

$$\begin{aligned} & \frac{\left(M^\ell + a^\ell\right)\frac{\partial w\left(e^\ell, z^\ell\right)}{\partial e^\ell}}{\left(w\left(e^\ell, z^\ell\right)\right)^2} v'\left(\frac{M^\ell + a^\ell}{w\left(e^\ell, z^\ell\right)}\right) \\ =\ & \lambda\left[\frac{\left(M^\ell + a^{h\ell}\right)\frac{\partial w\left(e^\ell, z^{h\ell}\right)}{\partial e^\ell}}{\left(w\left(e^\ell, z^{h\ell}\right)\right)^2} v'\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right)\right] + \mu\pi p, \end{aligned} \tag{C10}$$

$$\frac{v'\left(\frac{M^\ell + a^\ell}{w\left(e^\ell, z^\ell\right)}\right)}{w\left(e^\ell, z^\ell\right)} = \lambda \frac{v'\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right)}{w\left(e^\ell, z^{h\ell}\right)} + \mu\pi. \tag{C11}$$

Dividing (C10) by (C11), and multiplying both sides of the resulting equation by the RHS of (C11) gives

$$\begin{aligned} & \frac{\left(M^\ell + a^\ell\right)\frac{\partial w\left(e^\ell, z^\ell\right)}{\partial e^\ell}}{\left(w\left(e^\ell, z^\ell\right)\right)} \left[\lambda \frac{v'\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right)}{w\left(e^\ell, z^{h\ell}\right)} + \mu\pi\right] \\ =\ & \lambda\left[\frac{\left(M^\ell + a^{h\ell}\right)\frac{\partial w\left(e^\ell, z^{h\ell}\right)}{\partial e^\ell}}{\left(w\left(e^\ell, z^{h\ell}\right)\right)^2} v'\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right)\right] + \mu\pi p. \end{aligned}$$

Rearranging terms, the equation above can be rewritten as

$$L^\ell \frac{\partial w\left(e^\ell, z^\ell\right)}{\partial e^\ell} - p = \frac{\lambda}{\mu\pi} \frac{v'\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right)}{w\left(e^\ell, z^{h\ell}\right)} \left[L^{h\ell} \frac{\partial w\left(e^\ell, z^{h\ell}\right)}{\partial e^\ell} - L^\ell \frac{\partial w\left(e^\ell, z^\ell\right)}{\partial e^\ell}\right],$$

or equivalently,

$$L^\ell \frac{\partial w\left(e^\ell, z^\ell\right)}{\partial e^\ell} - p = \frac{\lambda}{\mu\pi} \frac{v'\left(\frac{M^\ell + a^{h\ell}}{w\left(e^\ell, z^{h\ell}\right)}\right)}{e^\ell w\left(e^\ell, z^{h\ell}\right)} \left(I^{h\ell} \epsilon_{w,e}^{h\ell} - I^\ell \epsilon_{w,e}^\ell\right). \tag{C12}$$

Under the assumption that the function $w(e, z)$ is Cobb-Douglas we have that $\epsilon_{w,e}^{h\ell} = \epsilon_{w,e}^\ell$ (despite the fact that $z^{h\ell} > z^\ell$).[29] Thus, from (C12) it follows that $L^\ell \frac{\partial w\left(e^\ell, z^\ell\right)}{\partial e^\ell} - p = 0$ in a

[29] Letting $w(e, z)$ be equal to $Ae^\varsigma z^\xi$ we have that $\epsilon_{w,e}^{h\ell} = \epsilon_{w,e}^\ell = \varsigma$.

setting without IM (where $I^{h\ell} = I^{\ell} = M^{\ell}$). Instead, in a setting with IM, we will have that

$$L^{\ell} \frac{\partial w\left(e^{\ell}, z^{\ell}\right)}{\partial e^{\ell}} - p = \frac{\lambda}{\mu \pi} \frac{v'\left(\frac{M^{\ell}+a^{h\ell}}{w\left(e^{\ell}, z^{h\ell}\right)}\right)}{e^{\ell} w\left(e^{\ell}, z^{h\ell}\right)} \left(I^{h\ell} - I^{\ell}\right) \epsilon_{w,e}^{\ell} > 0,$$

where the sign follows from the fact that (C9) remains positive even when $\frac{\partial w}{\partial \theta} = \frac{\partial^2 w}{\partial z \partial \theta} = 0$, which implies that $da/d\theta > 0$ and therefore $I^{h\ell} > I^{\ell}$.

References

[1] Allen, Stephen P., 1987. Taxes, redistribution, and the minimum wage: a theoretical analysis. Quarterly Journal of Economics 102, 477–490.

[2] Allingham, Michael G., and Agnar Sandmo. 1972. Income tax evasion: a theoretical analysis. Journal of Public Economics 1, 323–338.

[3] Aronsson, Thomas, and Tomas Sjögren, 2003. Income taxation, commodity taxation, and provision of public goods under labor market distortions. FinanzArchiv 59, 347–370.

[4] Aronsson, Thomas, and Tomas Sjögren, 2004. Is the optimal labor income tax progressive in a unionized economy? Scandinavian Journal of Economics 106, 661–675.

[5] Aronsson, Thomas, and Luca Micheletto, 2021. Optimal redistributive income taxation and efficiency wages. Scandinavian Journal of Economics 123, 3–32.

[6] Arrow, Kenneth, 1971. The utilitarian approach to the concept of equality in public expenditure. Quarterly Journal of Economics 85, 409–415.

[7] Atkinson, Anthony B., and Nicholas H. Stern, 1974. Pigou, taxation and public goods. Review of Economic Studies 41, 119–128.

[8] Atkinson, Anthony B., and Joseph E. Stiglitz, 1976. The design of tax structure: direct versus indirect taxation. Journal of Public Economics 6, 55-75.

[9] Bastani, Spencer, Blumkin, Tomer, and Luca Micheletto, 2015. Optimal wage redistribution in the presence of adverse selection in the labor market. Journal of Public Economics 131, 41–57.

[10] Blackorby, Charles, and Craig Brett, 2004. Production efficiency and the direct-indirect tax mix. Journal of Public Economic Theory 6, 165–180.

[11] Blomquist, Sören, Christiansen, Vidar, and Luca Micheletto, 2010. Public provision of private goods and nondistortionary marginal tax rates. American Economic Journal: Economic Policy 2, 1-27.

[12] Blomquist, Sören, Christiansen, Vidar, and Luca Micheletto, 2016. Public provision of private goods, self-selection, and income tax avoidance. Scandinavian Journal of Economics 118, 666-692.

[13] Blumkin, Tomer, and Efraim Sadka, 2008. A case for taxing education. International Tax and Public Finance 15, 145–163.

[14] Boadway, Robin and Firouz Gahvari, 2006. Optimal taxation with consumption time as a leisure or labor substitute. Journal of Public Economics 90, 1851–1878.

[15] Boadway, Robin, Marchand, Maurice and Pierre Pestieau, 1994. Towards a Theory of Direct-Indirect Tax Mix. Journal of Public Economics 55, 71–88.

[16] Boadway, Robin, Marchand, Maurice and Pierre Pestieau, 2000. Redistribution with Unobservable Bequests: A Case for Taxing Capital Income. Scandinavian Journal of Economics 102, 253–6718

[17] Boadway, Robin, Marchand, Maurice, Pestieau, Pierre, and Maria del Mar Racionero, 2002. Optimal Redistribution with Heterogeneous Preferences for Leisure. Journal of Public Economic Theory 4, 475–498.

[18] Bovenberg, Arij L. and Bas Jacobs, 2005. Redistribution and education subsidies are Siamese twins. Journal of Public Economics 89, 2005–2035.

[19] Brett, Craig, and John Weymark, 2003. Financing education using optimal redistributive taxation. Journal of Public Economics 87, 2549–2569.

[20] Bruno, Michael P., 1976. Equality, complementarity and the incidence of public expenditures. Journal of Public Economics 6, 395–407.

[21] Canta, Chiara, Cremer, Helmuth, and Firouz Gahvari, 2021. Welfare improving tax evasion. Mimeo.

[22] Chetty, Raj, 2009. Is the taxable income elasticity sufficient to calculate deadweight loss? The implications of evasion and avoidance. American Economic Journal: Economic Policy 1, 31–52.

[23] Christiansen, Vidar and Matti Tuomala, 2008. On taxing capital income with income shifting. International Tax and Public Finance 15, 527–545.

[24] Cremer, Helmuth, and Firouz Gahvari, 1993. Tax evasion and optimal commodity taxation. Journal of Public Economics 50, 261–275.

[25] Cremer, Helmuth, and Firouz Gahvari, 1994. Tax evasion, concealment, and the optimal linear income tax. Scandinavian Journal of Economics 96, 219–239.

[26] Cremer, Helmuth, and Firouz Gahvari, 1995a. Uncertainty, optimal taxation and the direct versus indirect tax controversy. Economic Journal 95, 1165–1179.

[27] Cremer, Helmuth, and Firouz Gahvari, 1995b. Uncertainty and optimal taxation: in defense of commodity taxes. Journal of Public Economics 56, 291–310.

[28] Cremer, Helmuth, and Firouz Gahvari, 1996. Tax evasion and the optimum general income tax. Journal of Public Economics 60, 235–249.

[29] Cremer, Helmuth, and Firouz Gahvari, 1998. On optimal taxation of housing. Journal of Urban Economics 43, 315–335.

[30] Cremer, Helmuth, and Firouz Gahvari, 1999. Uncertainty, commitment and optimal taxation. Journal of Public Economic Theory 1, 51–70.

[31] Cremer, Helmuth, and Firouz Gahvari, 2002. Nonlinear pricing, redistribution and optimal tax policy. Journal of Public Economic Theory 4, 139–161.

[32] Cremer, Helmuth, and Firouz Gahvari, 2015. Atkinson and Stiglitz theorem in the presence of a household production sector, Economics Letters 126, 91–95.

[33] Cremer, Helmuth, Marchand, Maurice, and Pierre Pestieau, 1990. Evading, auditing and taxing: The equity-compliance tradeoff. Journal of Public Economics 43, 67–92.

[34] Cremer, Helmuth, Pestieau, Pierre, and Jean-Charles Rochet, 2001. Direct versus Indirect Taxation: The Design of the Tax Structure Revisted. International Economic Review 42, 781–799.

[35] Cremer, Helmuth, Pierre Pestieau and Jean-Charles Rochet, 2002. Capital income taxation when inherited wealth is not observable. Journal of Public Economics 87, 2475–2490.

[36] da Costa, Carlos, and Lucas J. Maestri, 2007. The risk properties of human capital and the design of government policies. European Economic Review 51, 695–713.

[37] da Costa, Carlos, and Tiago Severo, 2008. Education, preferences for leisure and the optimal income tax schedule. Journal of Public Economics 92, 113–138.

[38] Findeisen, Sebastian, and Dominik Sachs, 2016. Education and optimal dynamic taxation: the role of income-contingent student loans. Journal of Public Economics 138, 1–21.

[39] Findeisen, Sebastian, and Dominik Sachs, 2018. Education policies and taxation wiyhout commitment. Scandinavian Journal of Economics 120, 1075–1099.

[40] Gahvari, Firouz, 2007. On optimal commodity taxes when consumption is time consuming. Journal of Public Economic Theory 9, 1–27.

[41] Gahvari, Firouz, 2014. Second-best taxation of incomes and non-labor inputs in a model with endogenous wages, Journal of Public Economic Theory 16, 917–935.

[42] Gahvari, Firouz, and Luca Micheletto, 2014. The Friedman rule in an overlapping-generations model with nonlinear taxation and income misreporting. Journal of Public Economics 119, 10–23.

[43] Gahvari, Firouz, and Luca Micheletto, 2016. Capital income taxation and the Atkinson–Stiglitz theorem. Economics Letters 147, 86–89.

[44] Gaube, Thomas, 2005. Income taxation, endogenous factor prices and production efficiency. Scandinavian Journal of Economics 107, 335–352.

[45] Gerritsen, Aart, 2021. Income tax evasion and third-party reported consumption expenditures: implications for the optimal tax structure. Mimeo.

[46] Green, Jerry R., and Eytan Sheshinski, 1975. A note on the progressivity of optimal public expenditures. Quarterly Journal of Economics 89, 138-144.

[47] Guo, Jang-Ting, and Alan Krause, 2013. Optimal nonlinear taxation of income and education expenditures. Oxford Economic Papers 65, 74-95.

[48] Hare, Paul G., and David T. Ulph, 1979. On education and distribution. Journal of Political Economy 87, S193-S212.

[49] Hellwig, Martin, 2008. A generalization of the Atkinson-Stiglitz (1976) theorem on the undesirability of nonuniform excise taxation. Discussion Paper Series of the Max Planck Institute for Research on Collective Goods 2008_47.

[50] Hungerbühler, Mathias, Lehmann, Etienne, Parmentier, Alexis, and Bruno Van Der Linden, 2006. Optimal redistributive taxation in a search equilibrium model. Review of Economic Studies 73, 743–767.

[51] Jacobs, Bas, and Arij L. Bovenberg, 2011. Optimal taxation of human capital and the earnings function. Journal of Public Economic Theory 13, 957-971.

[52] Kapicka, Marek, and Julian Neira, 2019. Optimal taxation with risky human capital. American Economic Journal: Macroeconomics 11, 271–309.

[53] Kaplow, Louis, 1990. Optimal taxation with costly enforcement and evasion. Journal of Public Economics 43, 221–236.

[54] Kessing, Sebastian, and Kai Konrad, 2006. Union strategy and optimal direct taxation. Journal of Public Economics 90, 393–402.

[55] Kopczuk, Wojciech, 2001. Redistribution when avoidance behavior is heterogeneous. Journal of Public Economics 81, 51–71.

[56] Mayshar, Joram, 1991. Taxation with costly administration. Scandinavian Journal of Economics 93, 75–88.

[57] Maldonado, Dario, 2008. Education polices and optimal taxation. International Tax and Public Finance 15, 131–143.

[58] Mirrlees, James A., 1971. An exploration in the theory of optimum income taxation. Review of Economic Studies 38, 175–208.

[59] Naito, Hisahiro, 1999. Re-examination of uniform commodity taxes under a non-linear tax system and its implications for production efficiency. Journal of Public Economics 71, 165–188.

[60] Pirttilä, Jukka, and Matti Tuomala, 2002. Publicly provided private goods and redistribution: a general equilibrium analysis. Scandinavian Journal of Economics 104, 173–188.

[61] Saez, Emmanuel, 2002. The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes. Journal of Public Economics. 83, 217–230.

[62] Schroyen, Fred, 1997. Pareto efficient income taxation under costly monitoring. Journal of Public Economics 65, 343–366.

[63] Slemrod, Joel, 2001. A general model of the behavioral response to taxation. International Tax and Public Finance 8, 119–128.

[64] Stantcheva, Stefanie, 2017. Optimal taxation and human capital policies over the life cycle. Journal of Political Economy 125, 1931–1990.

[65] Stern, Nicholas H., 1982. Optimum taxation with errors in administration. Journal of Public Economics 17, 181–211.

[66] Stiglitz, Joseph E., 1982. Self-selection and Pareto efficient taxation. Journal of Public Economics 17, 213–240.

[67] Stiglitz, Joseph E., 1987. Pareto efficient and optimal taxation and the new new welfare economics. In: Handbook of Public Economics vol. 2 (edited by Alan Auerbach and Martin Feldstein), ch. 15, 991–1042, North-Holland.

[68] Tuomala, Matti, 1986. On the optimal income taxation and educational decisions. Journal of Public Economics 30, 183–198.

[69] Ulph, David, 1977. On the optimal distribution of income and educational expenditure. Journal of Public Economics 8, 341–356.

[70] Usher, Dan, 1986. Tax evasion and the marginal cost of public funds. Economic Inquiry 24, 563–86.